



# Signal processing applied to ocean waves: dissipation and nonlinear interactions

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Show the application of Fourier Transforms in the study of ocean waves

**“The smart, brute force”**

1

Justification of studying the transformation  
of ocean waves

# Sea level rise threatening Kennedy Space Center in Florida

ORLANDO, FLA. | BY BARBARA LISTON



BUSINESS INSIDER

SCIENCE

## Rising Sea Levels Are Destroying NASA's Multi-Billion Dollar Facilities

AGENCE FRANCE PRESSE  
May 20, 2014, 6:56 AM 6,377 15

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## NASA Battles Rising Sea Levels To Protect Kennedy Space Center

APRIL 06, 2015 5:14 PM ET

AMY GREEN

Beach sediment relocation is linked to **wave transformation** at inner-shelf shoals

### El mar está a 27 metros del volcán de Arboletes



La erosión sigue. El volcán está junto a los árboles. FOTO CORTESÍA

MEDIO AMBIENTE URABÁ

PUBLICADO EL 24 DE ENERO DE 2015

A+ A-

### El mar se come el litoral del golfo de Urabá

El volcán de lodo y Arboletes están en alto riesgo. Hacen estudios para determinar solución.

Por: Yeison Gualdrón |  
© 1:49 a.m. | 26 de enero de 2015



Foto: EL TIEMPO  
El turismo es la principal fuente de ingresos de Arboletes.

EN VIVO

Teleantioquia Noticias

You Tube



Inicio / Teleantioquia Noticias / ANTIOQUIA

## Preocupación por la erosión costera en el Urabá antioqueño

ES NOTICIA EN ESTE MOMENTO:

### Desaparecen los manglares del golfo de Urabá



RELACIONADO CON: Catástrofes Naturales Antioquia

El golfo de Urabá, el más extenso del Caribe colombiano, se halla amenazado ambientalmente según reveló el proyecto Expedición Estuarina golfo de Urabá.

Caracol Radio | 6 de Febrero de 2013





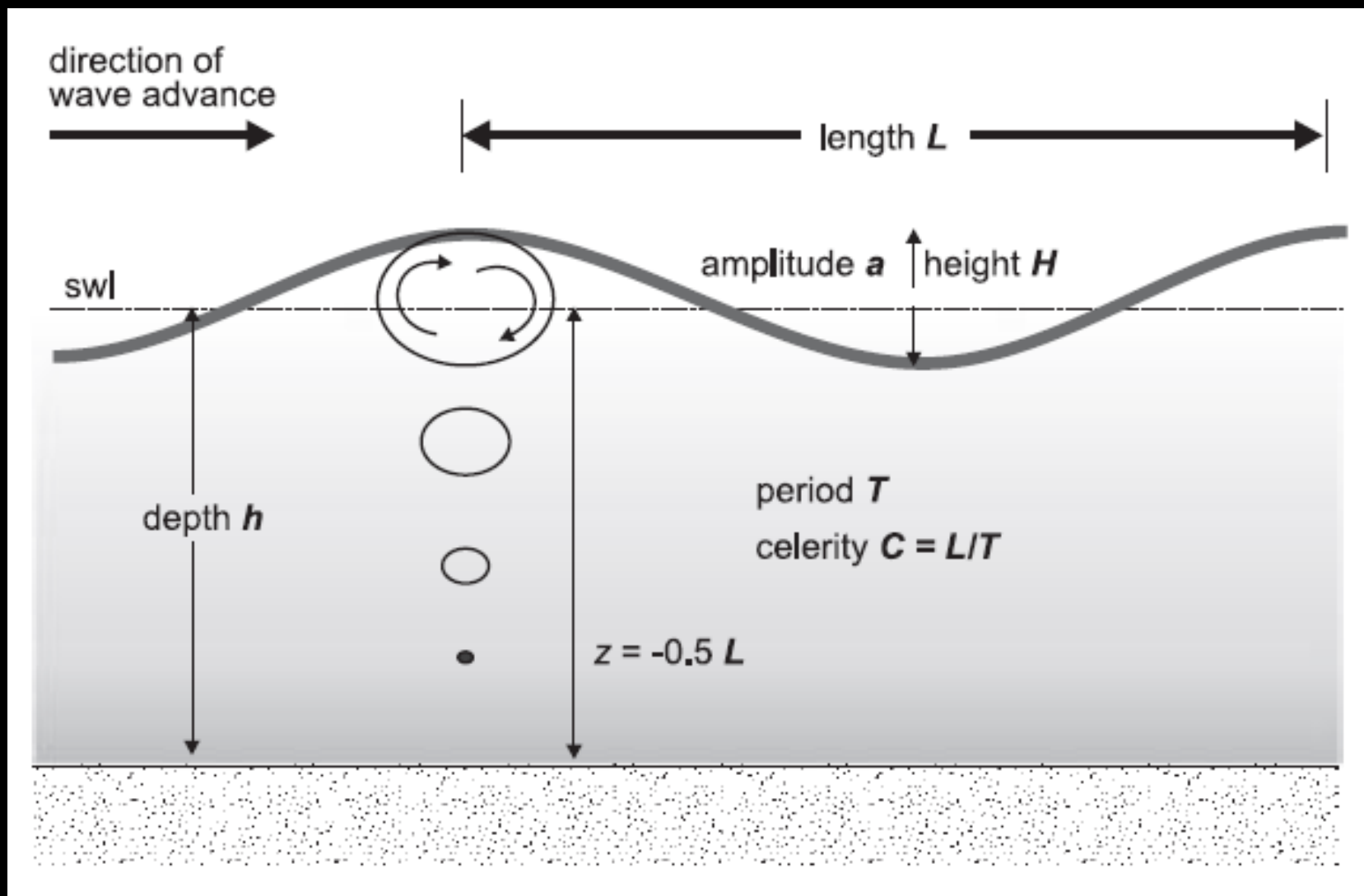
<http://data.whicdn.com/images/58661336/large.jpg>

# 2

## Frequency-dispersive waves

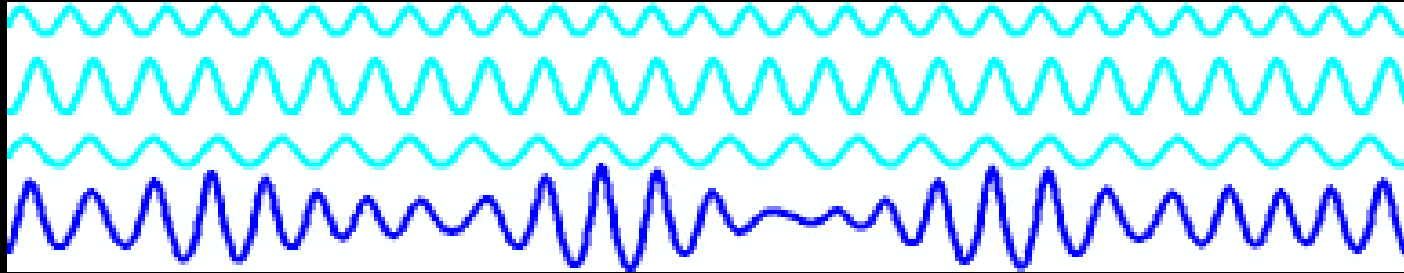


# SEA-SWELL -- 3 to 20 s

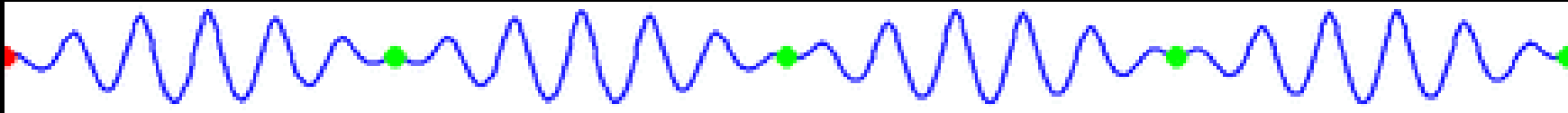


[1]

# SIMILAR PERIODS

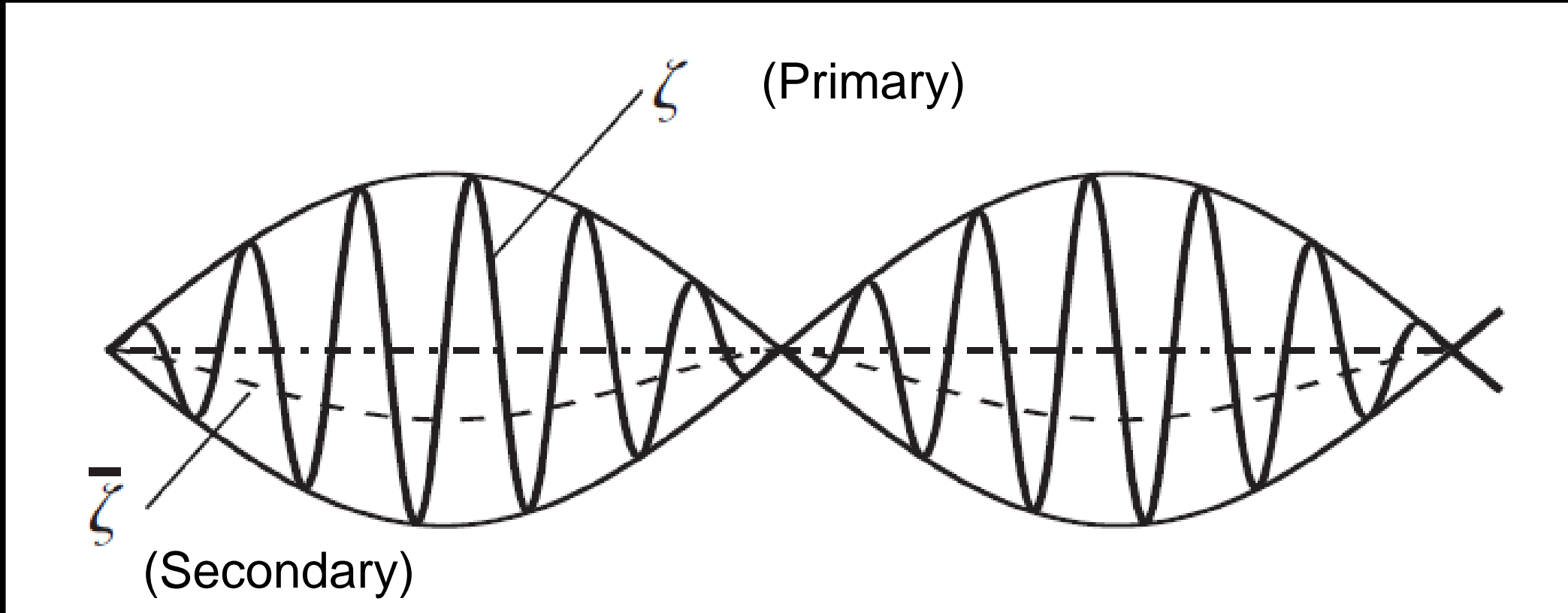


*Wikimedia Commons*



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# SURF BEAT – FORCED INFRAGRAVITY MOTIONS

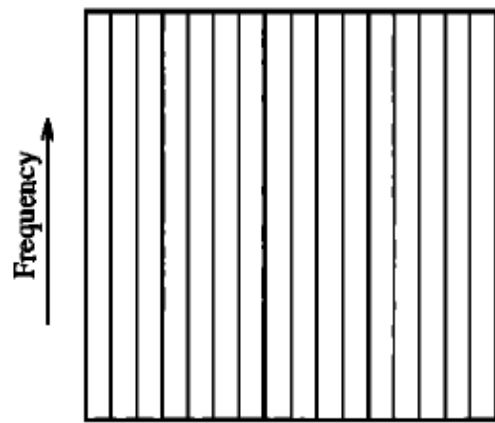


20 to 500 s

[2]

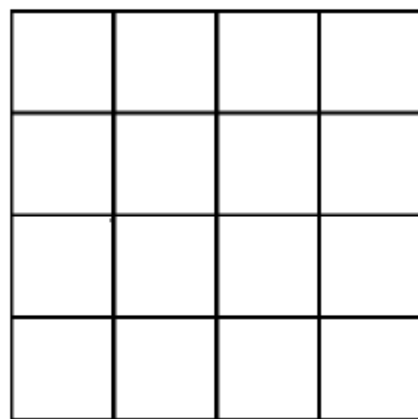
# 3

## Fourier series representation of ocean waves



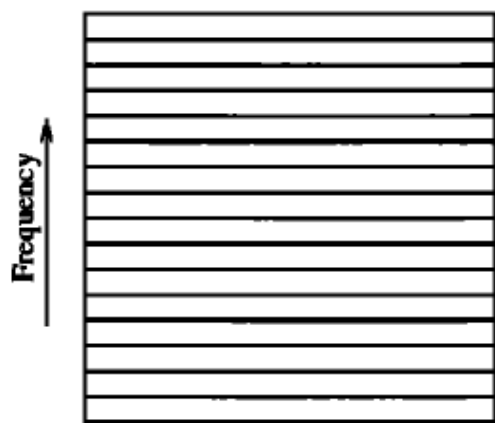
Time  
Standard basis

(a)



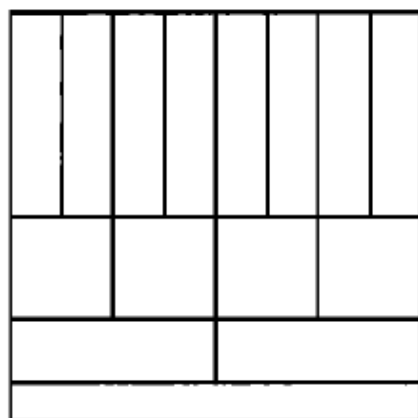
Time  
Windowed Fourier basis

(c)



Time  
Fourier basis

(b)

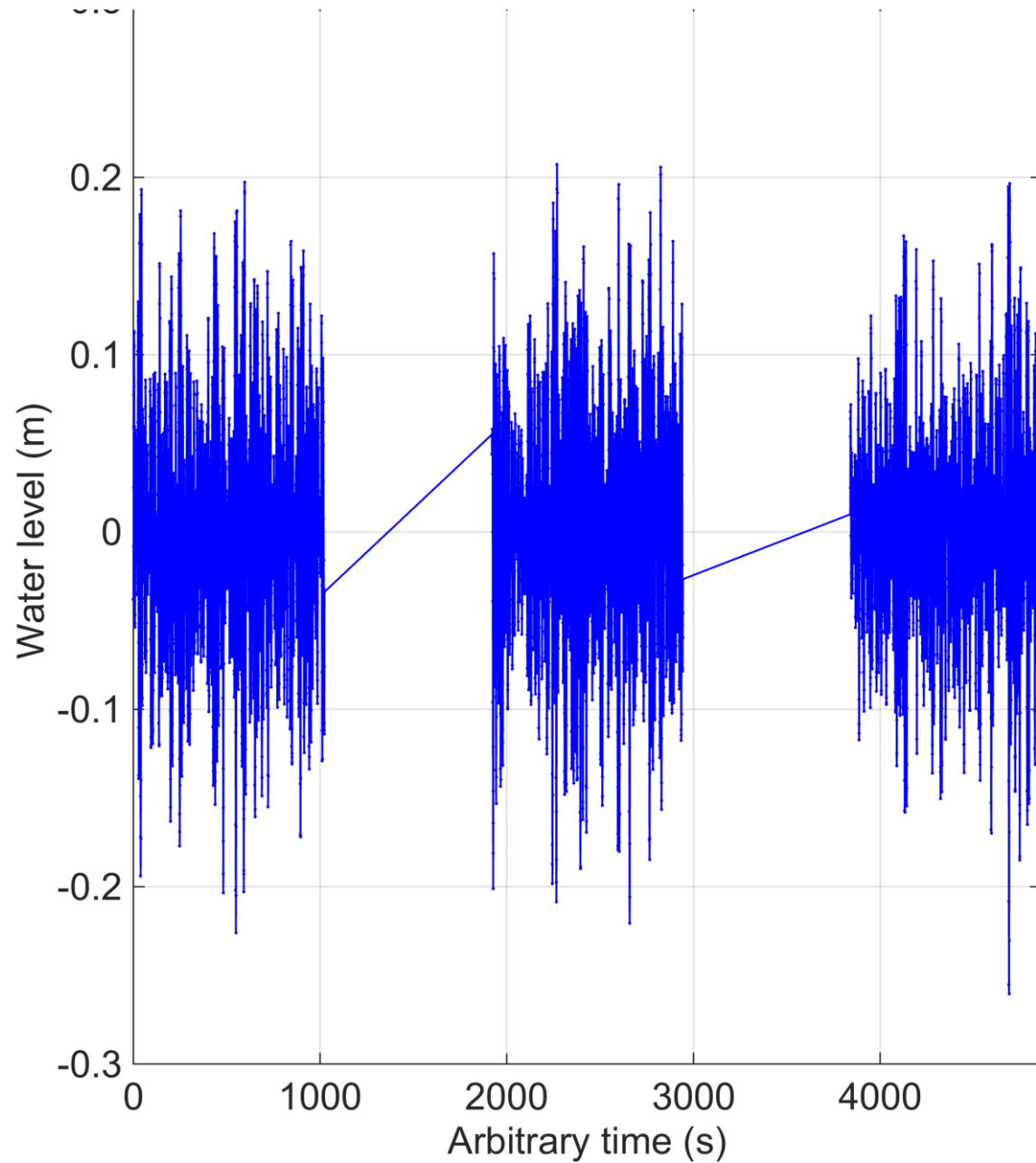


Time  
Wavelet basis

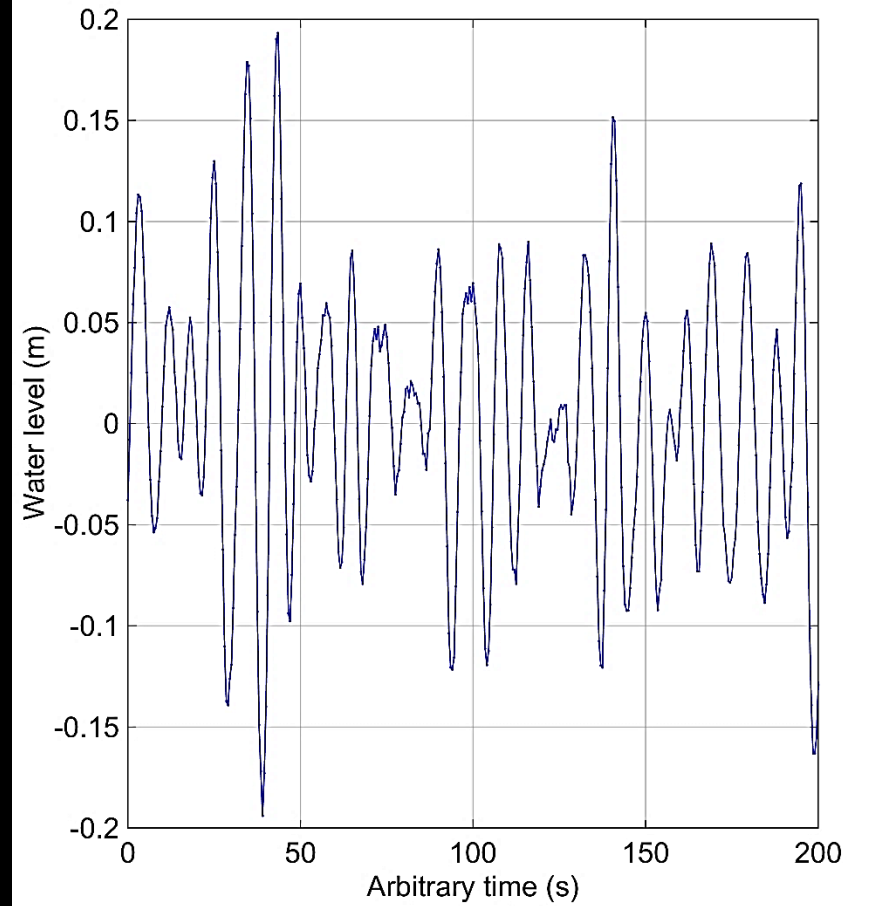
(d)

[3]

# WATER LEVEL



# WATER LEVEL





$$\eta(x, t) = \sum_{k=1}^{N-1} \left[ \hat{x}_k e^{i(\kappa_k x - \omega_k t)} + \hat{x}_k^* e^{-i(\kappa_k x - \omega_k t)} \right]$$

$$\omega_k^2 = g \kappa_k \tanh(\kappa_k h)$$

[4]

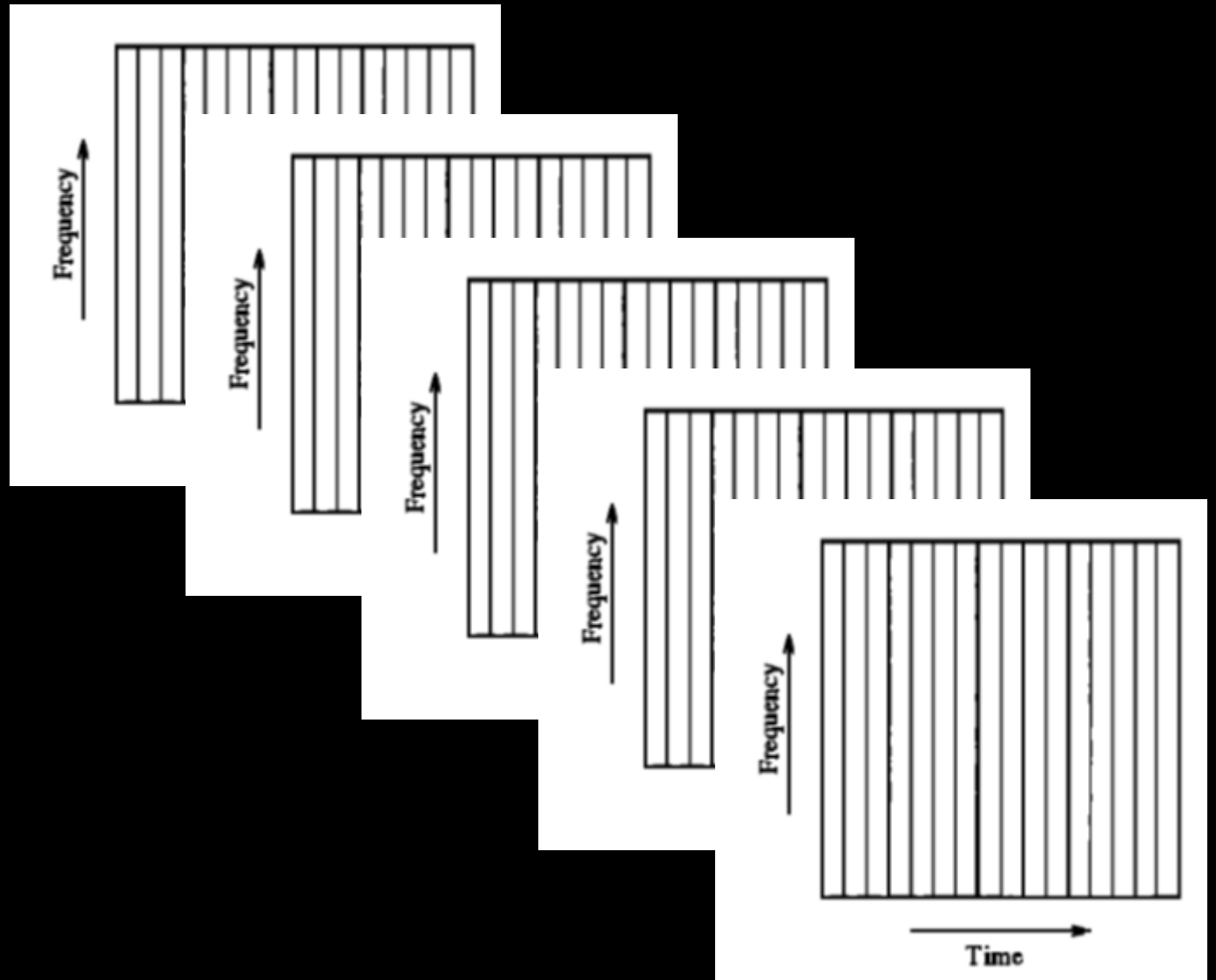
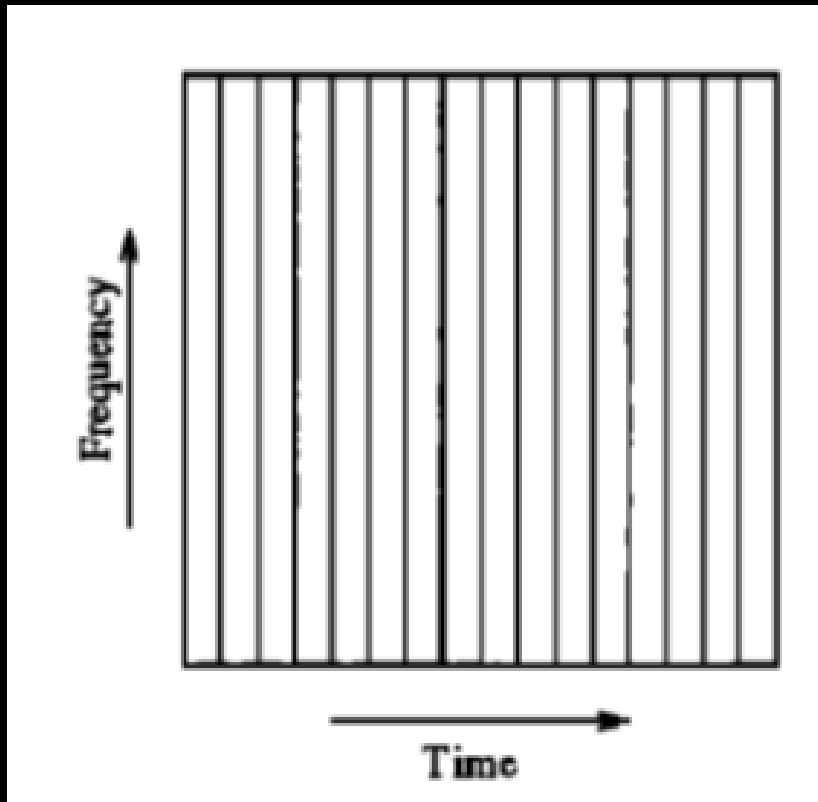
$$\hat{x}_k = \Delta t \sum_{n=1}^N x_n e^{-i2\pi f_k n \Delta t}$$

$$E(f_k) = \mathbb{E} [\hat{x}_k \hat{x}_k^*]$$

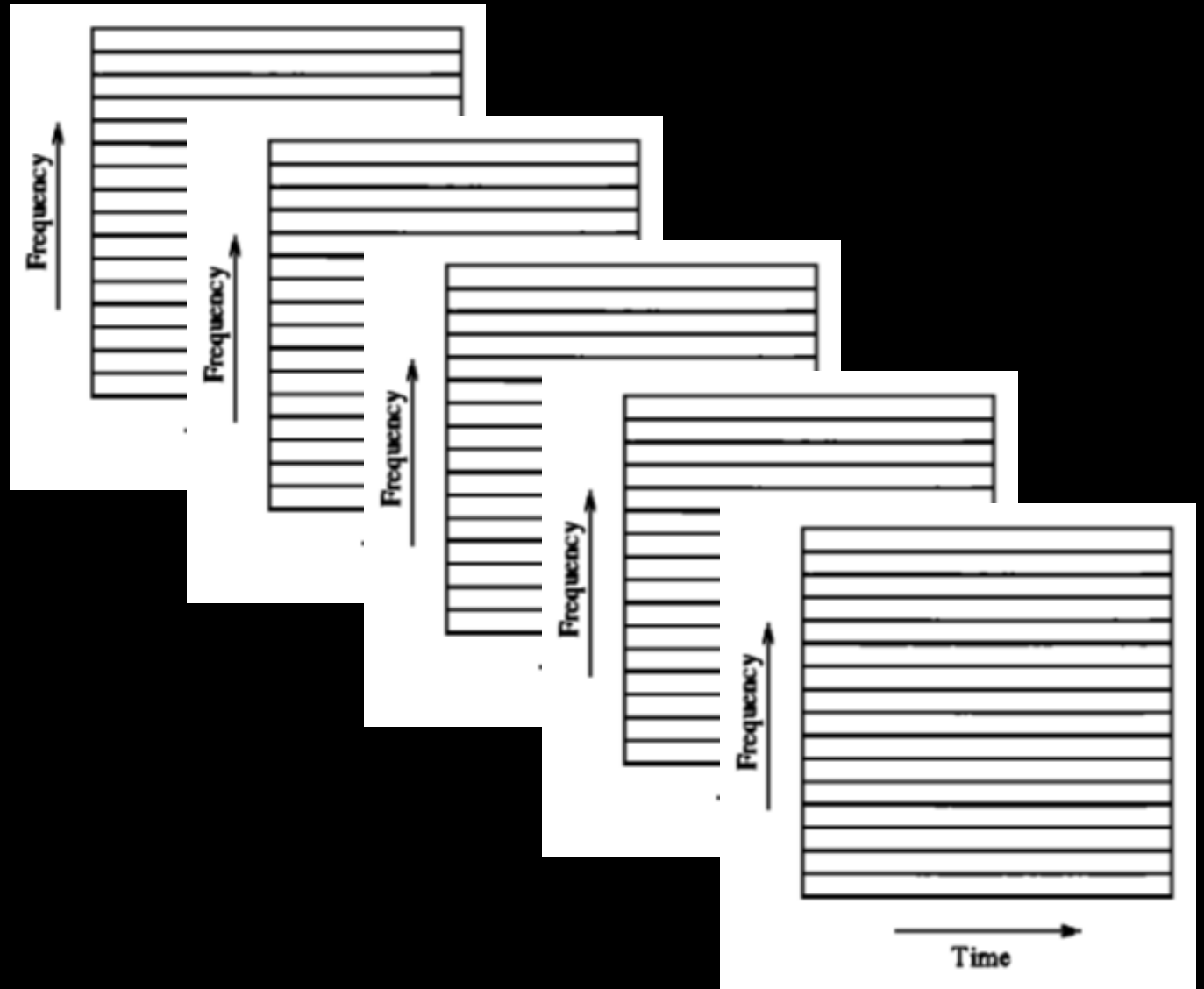
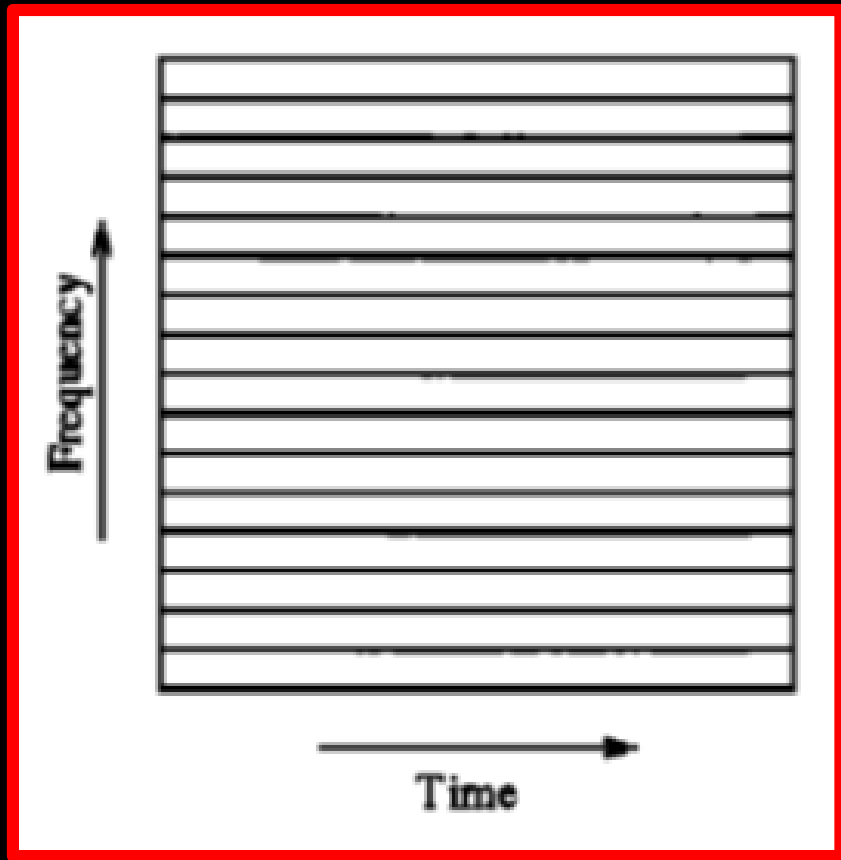
[5]

# TIME PLANE

$$x_n$$

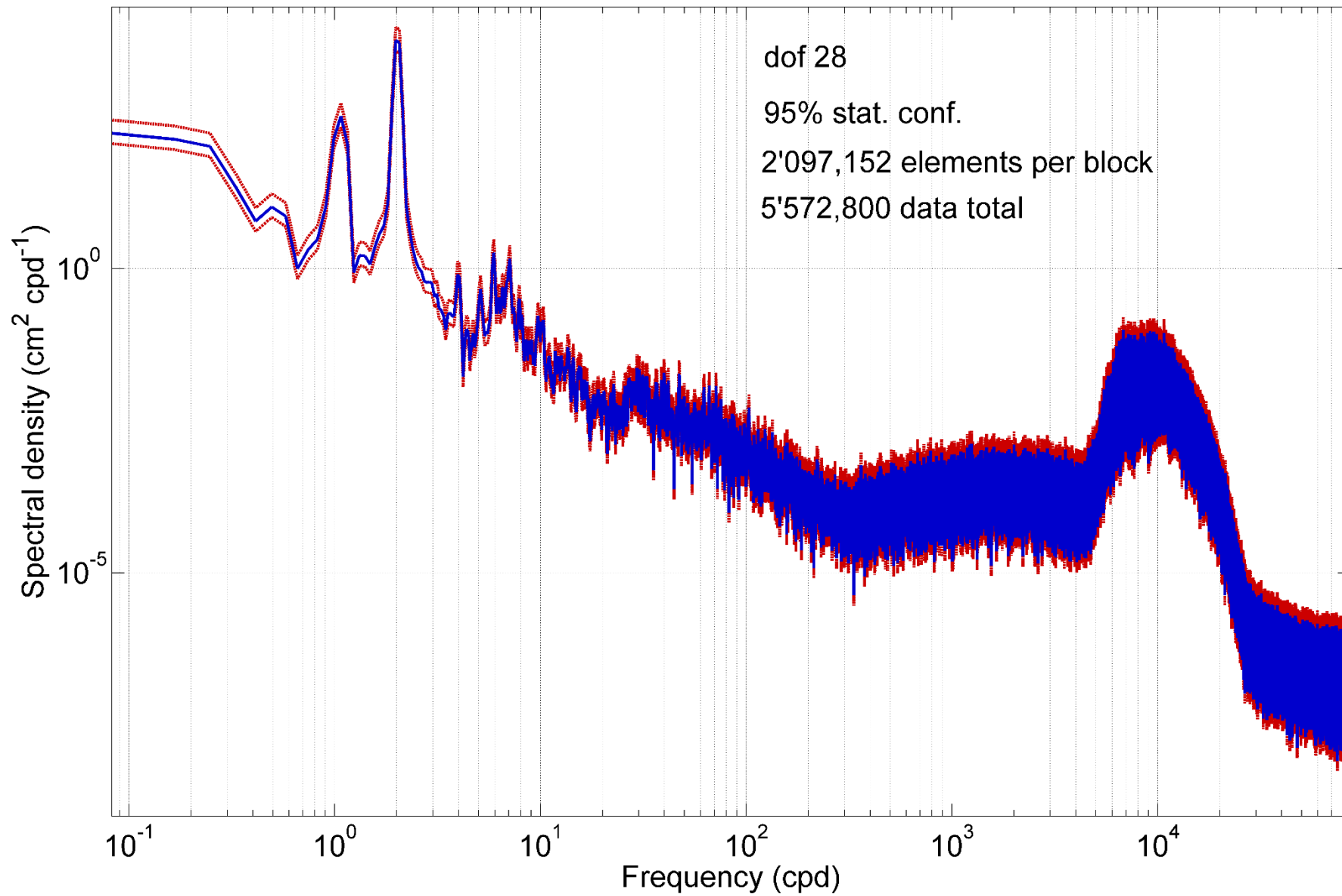


# FREQUENCY PLANE (Power spectrum)

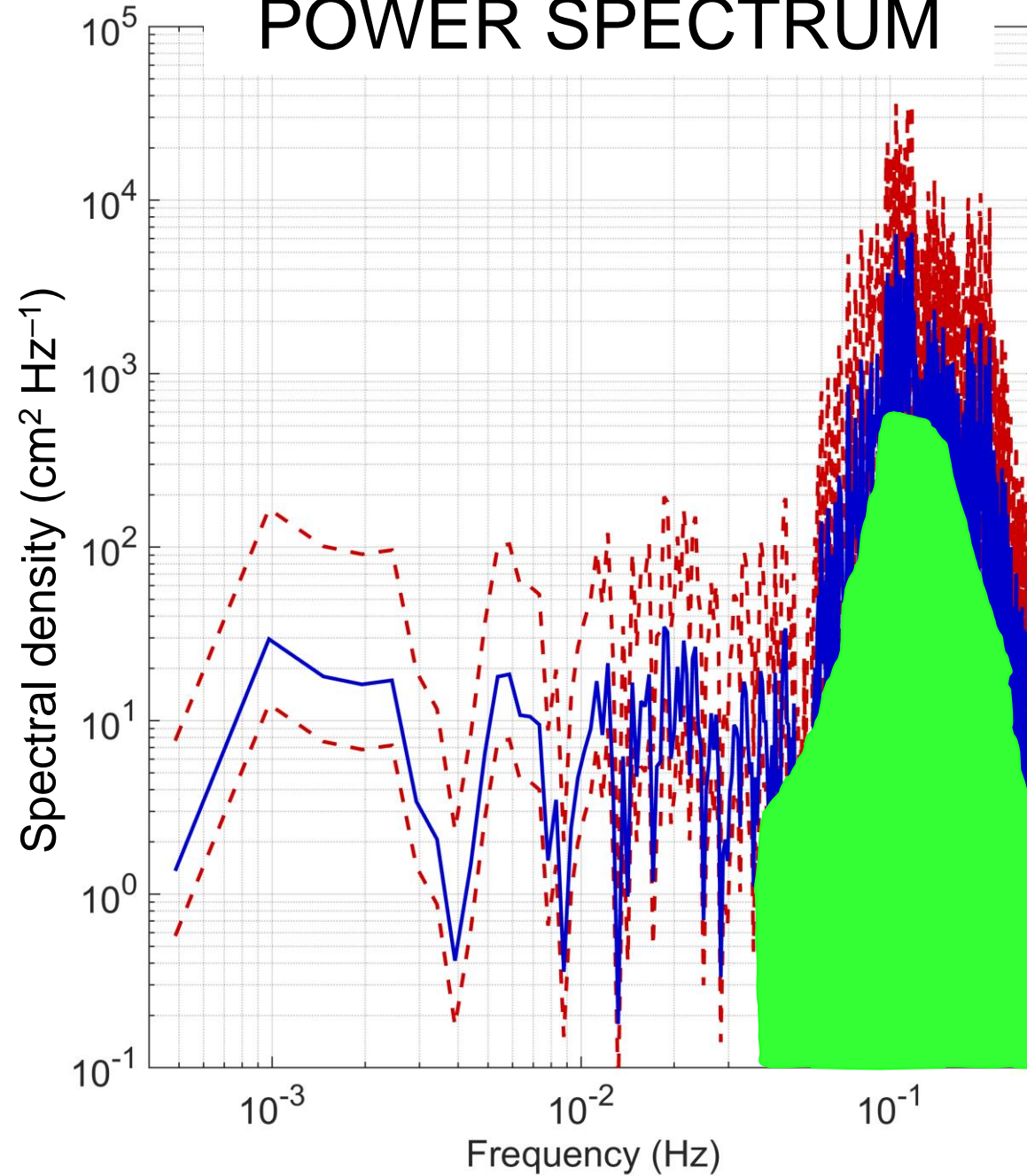


$$E(f_k) = E[\hat{x}_k \hat{x}_k^*]$$

Cape Canaveral shoals -- Shoal E inner swale, Spring 2015

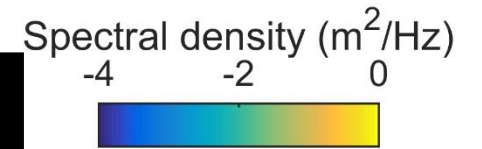
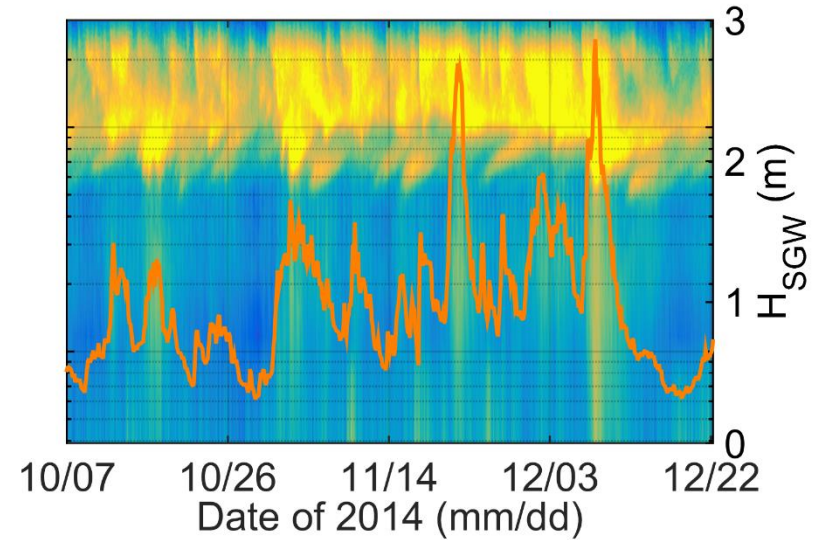
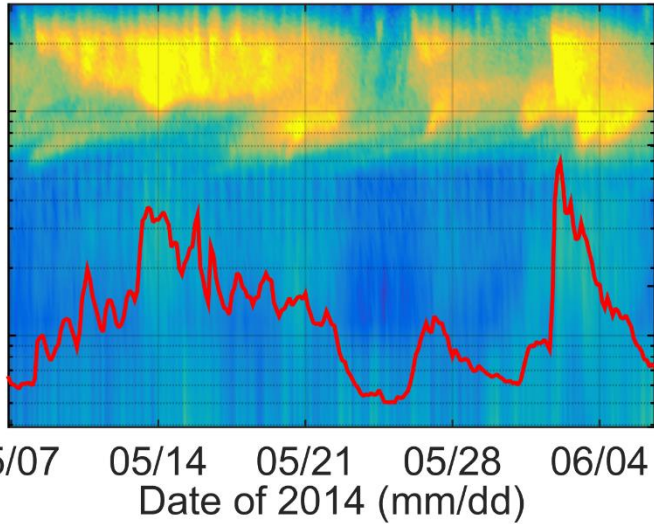
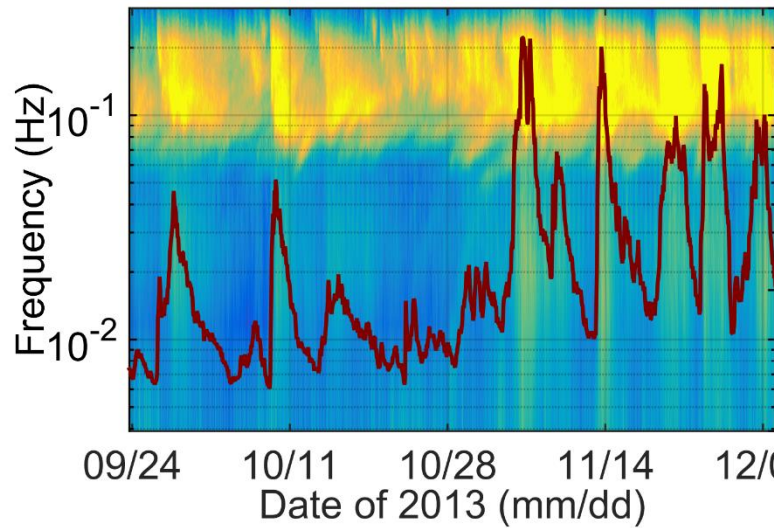


# POWER SPECTRUM



**VARIANCE**  
**Sea-swell**

# SPECTROGRAPHS





$$W_n(s) = \sum_{k=0}^{N-1} \hat{x}_k \hat{\psi}^*(s\omega_k) e^{i\omega_k n \Delta t}$$

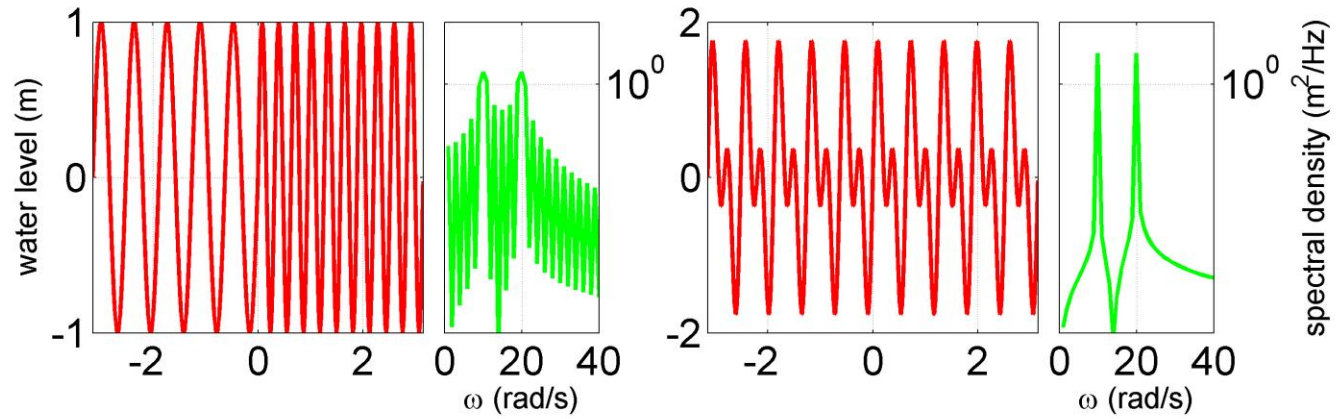
**SCALE  
(FREQUENCY)**

**FREQUENCY**

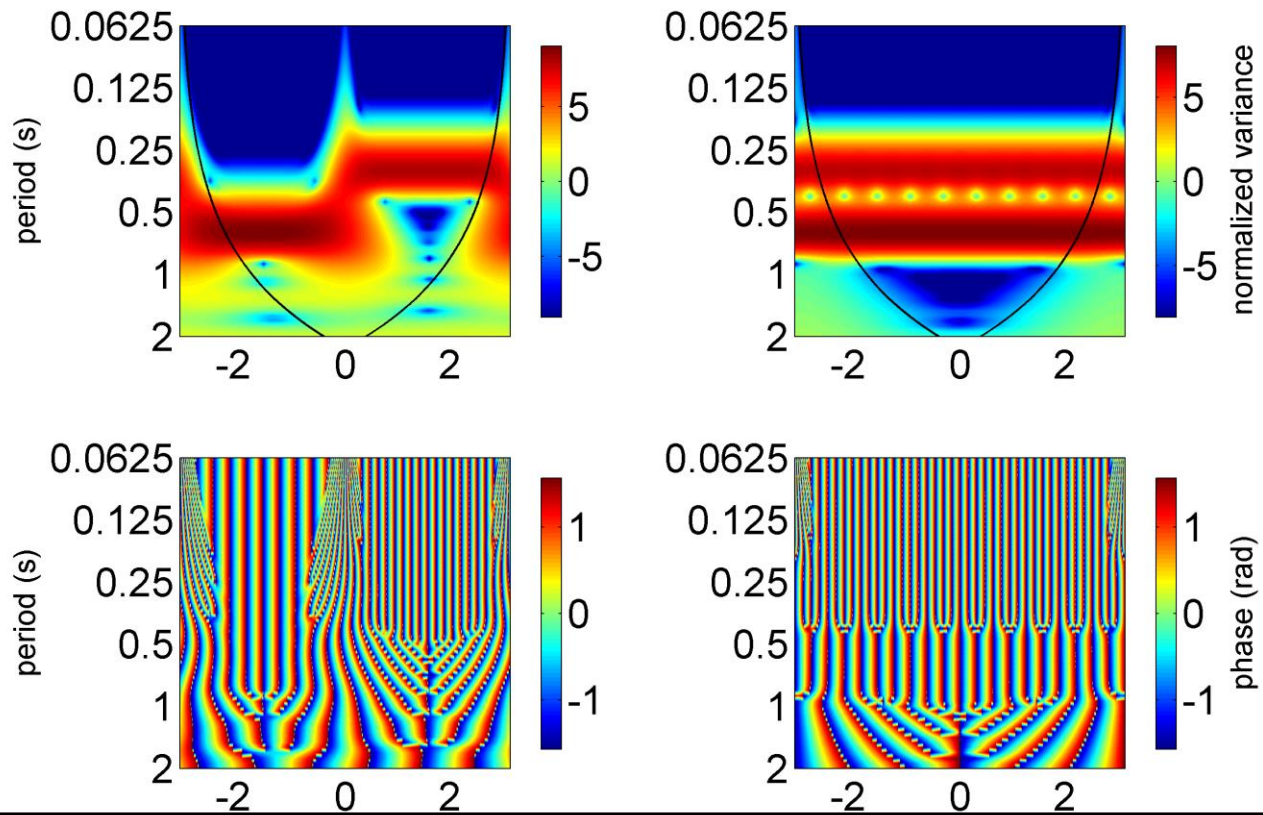
[6]

$$f_1 = \sin(10t)$$

$$f_2 = \sin(20t)$$



$$f_3 = \sin(10t) + \sin(20t)$$

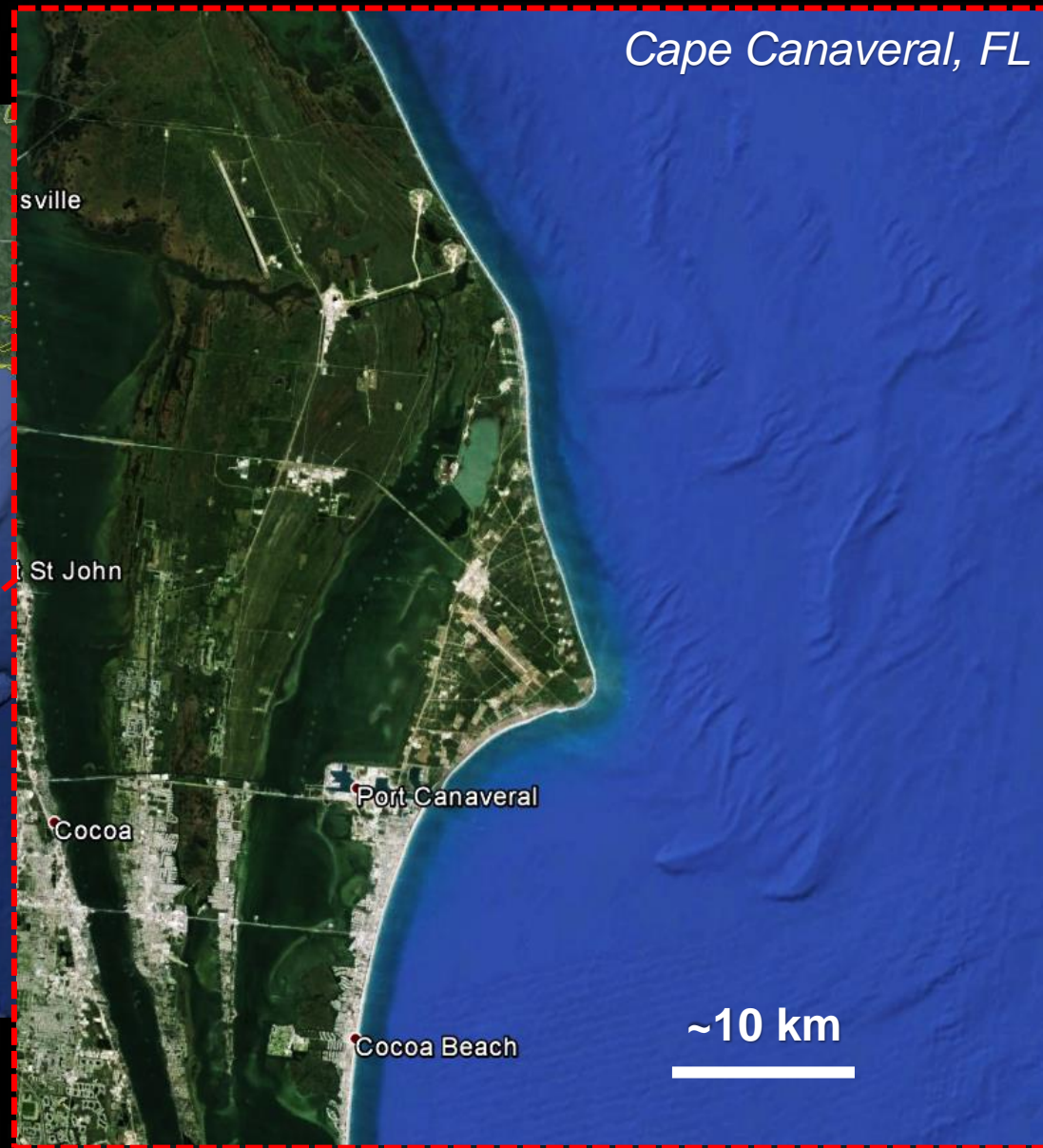
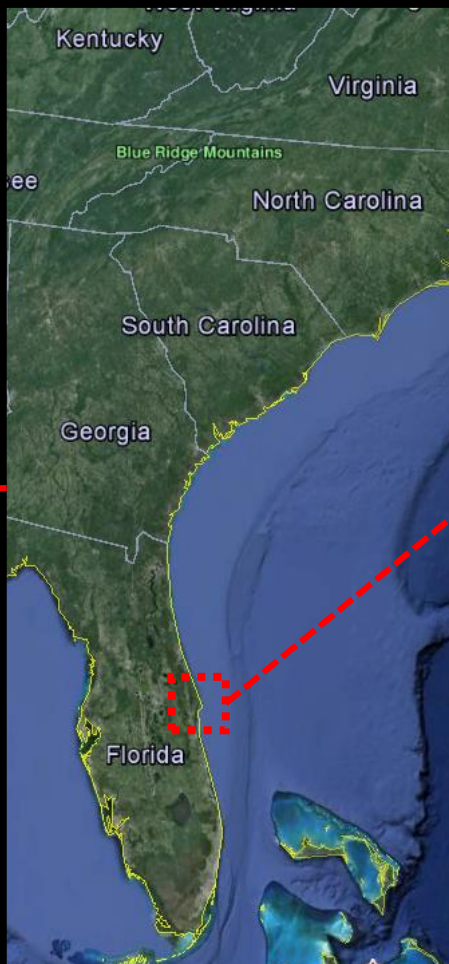


# 5

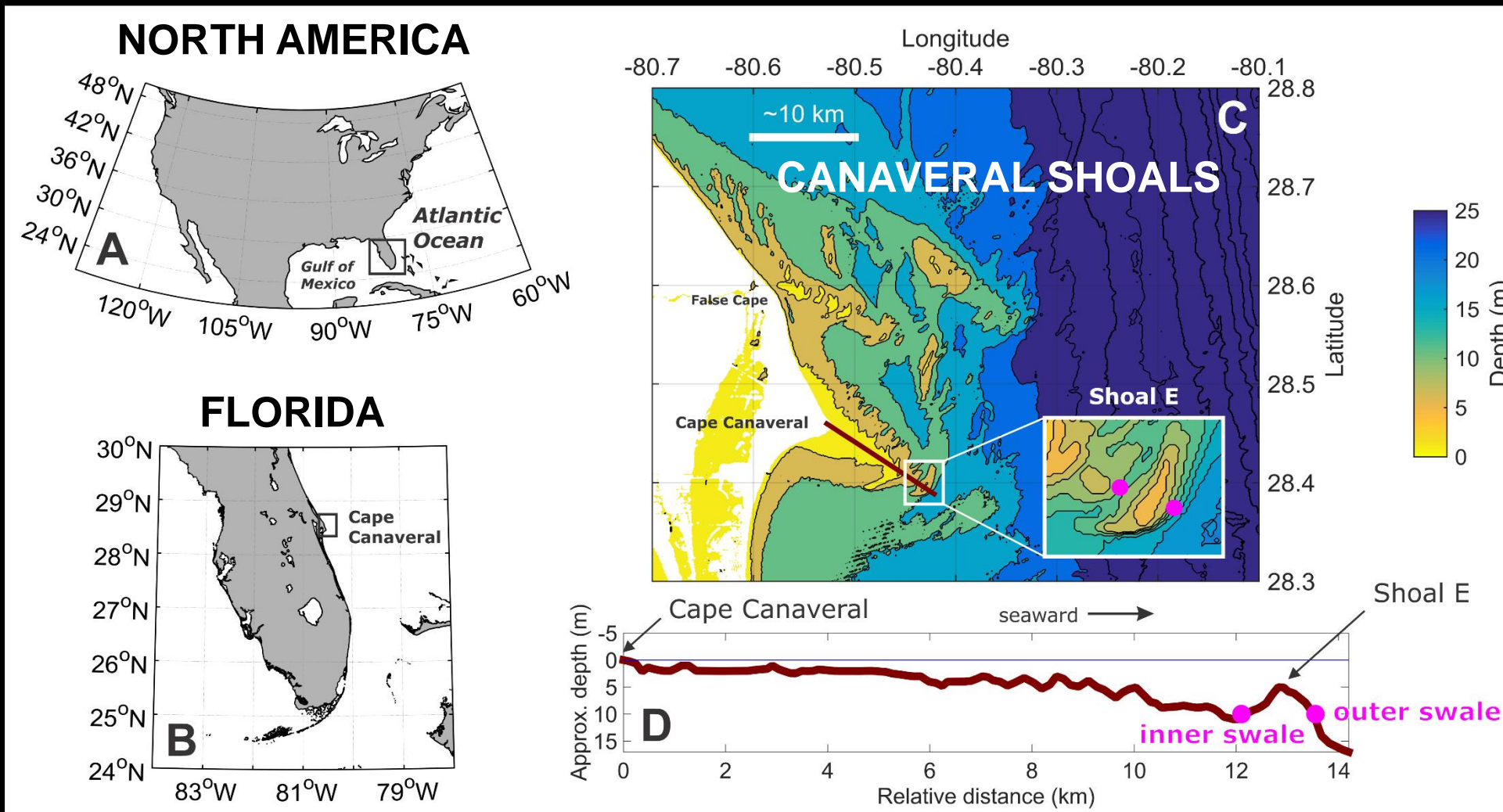
Application to wave transformation over  
Cape Canaveral shoals



US Dept of State Geographer  
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 Data SIO, NOAA, U.S. Navy, NGA, GEBCO



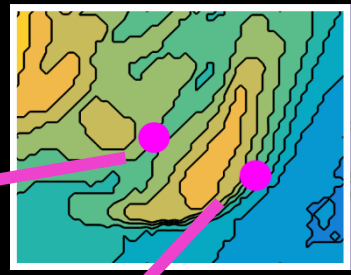




**ADCP**  
Acoustic Doppler  
Current Profiler with  
Pressure sensor

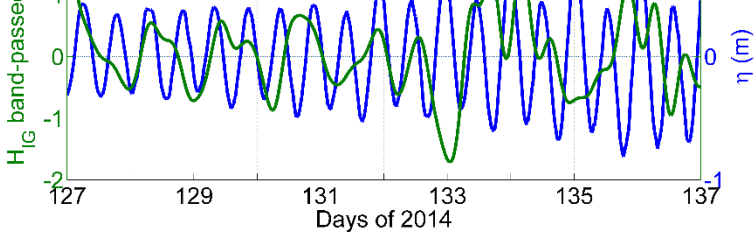
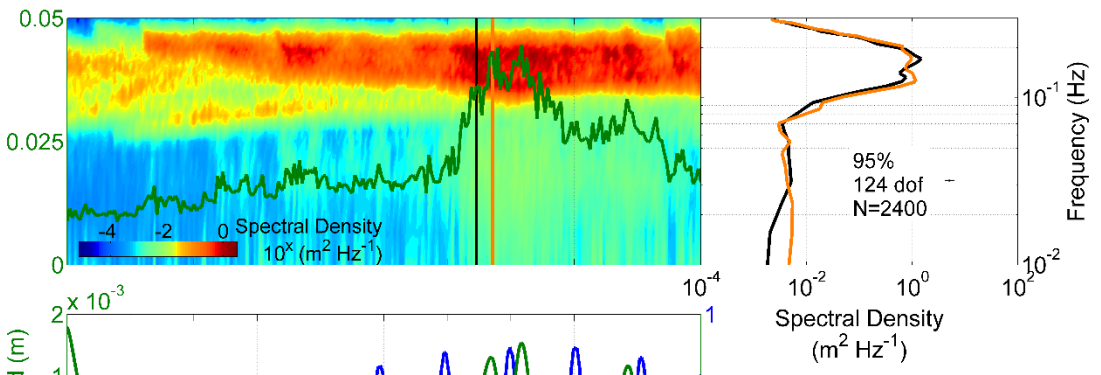
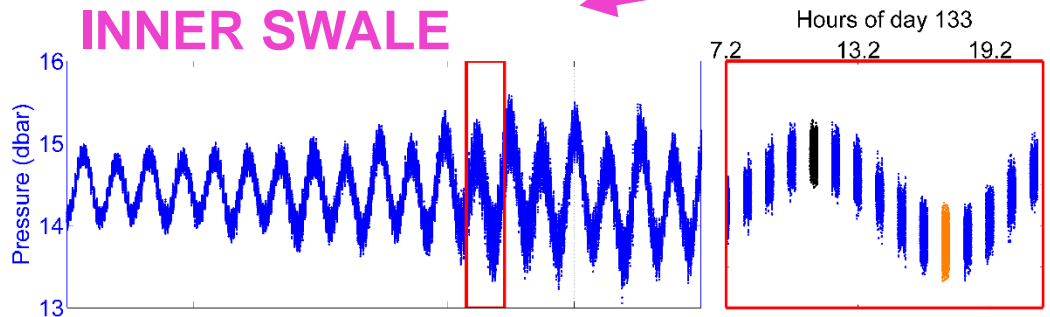


13 ADCP campaigns  
between  
Fall 2013 and  
Fall 2016

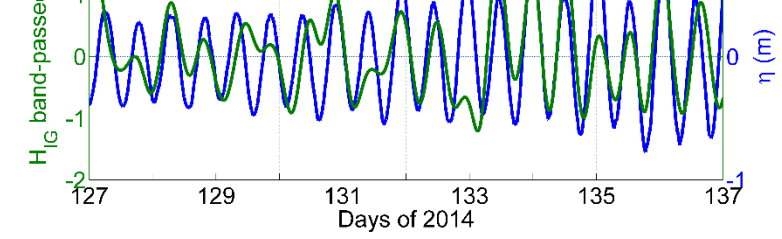
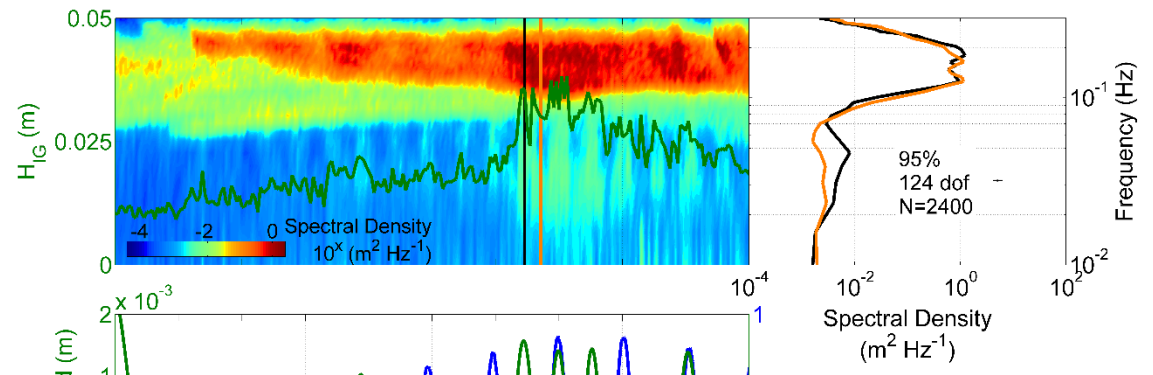
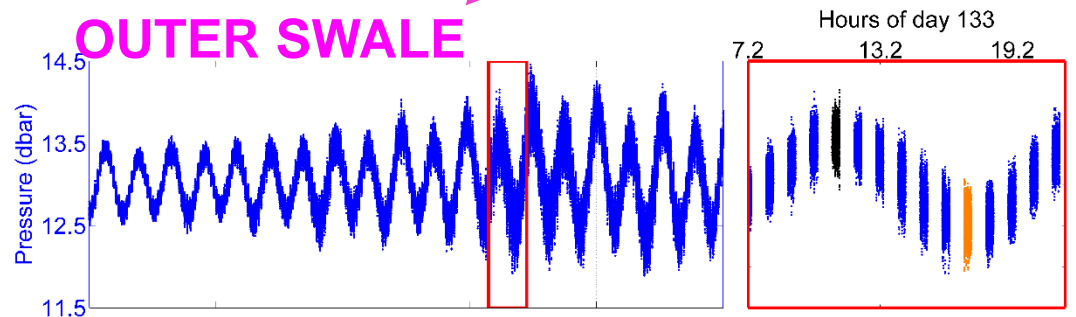


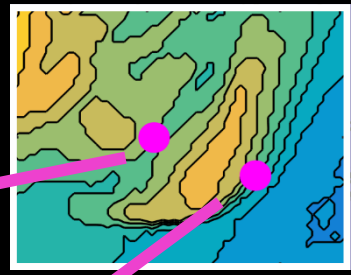
Shoal E

**INNER SWALE**

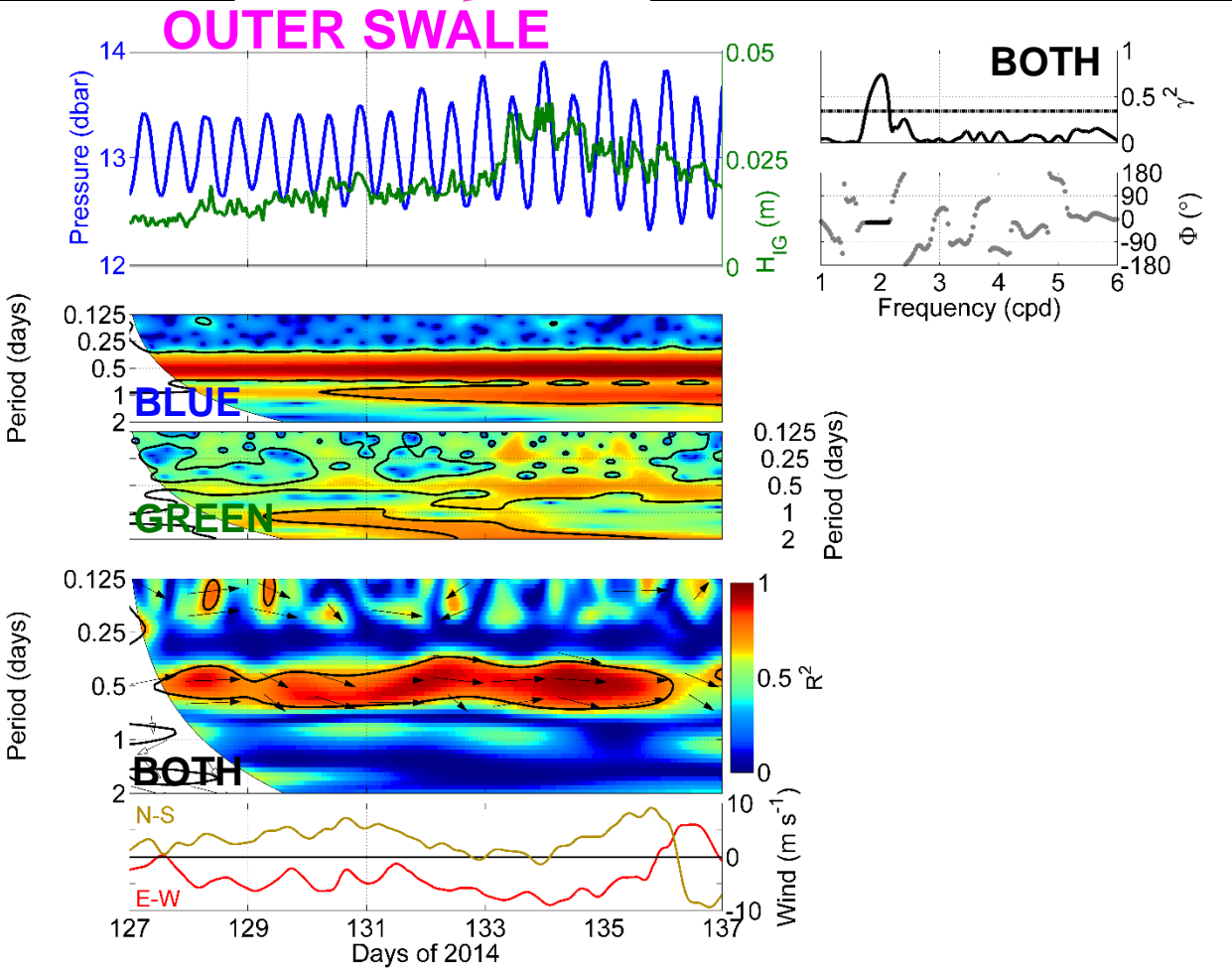
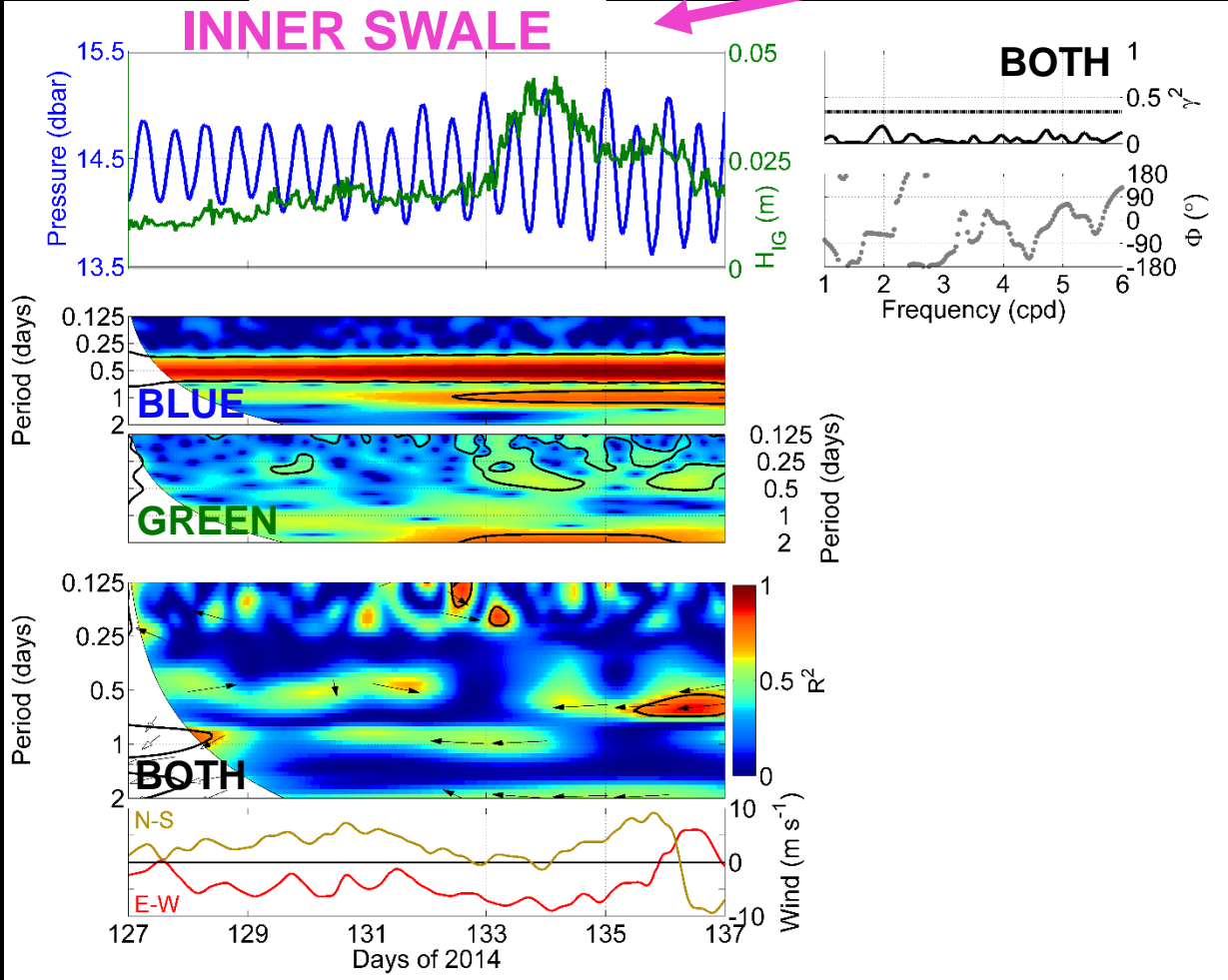


**OUTER SWALE**



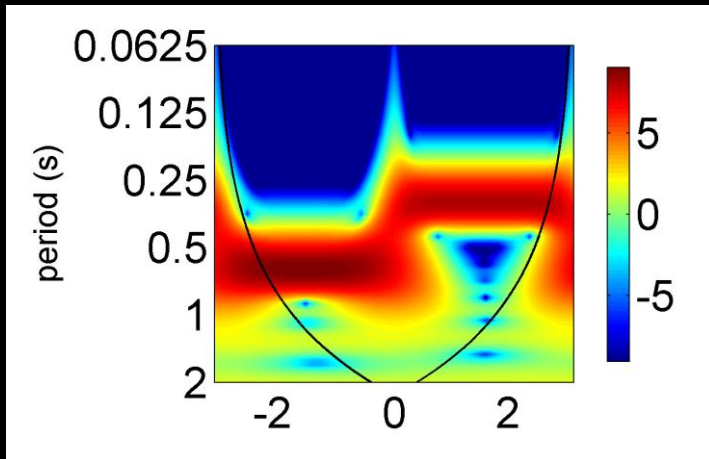


Shoal E







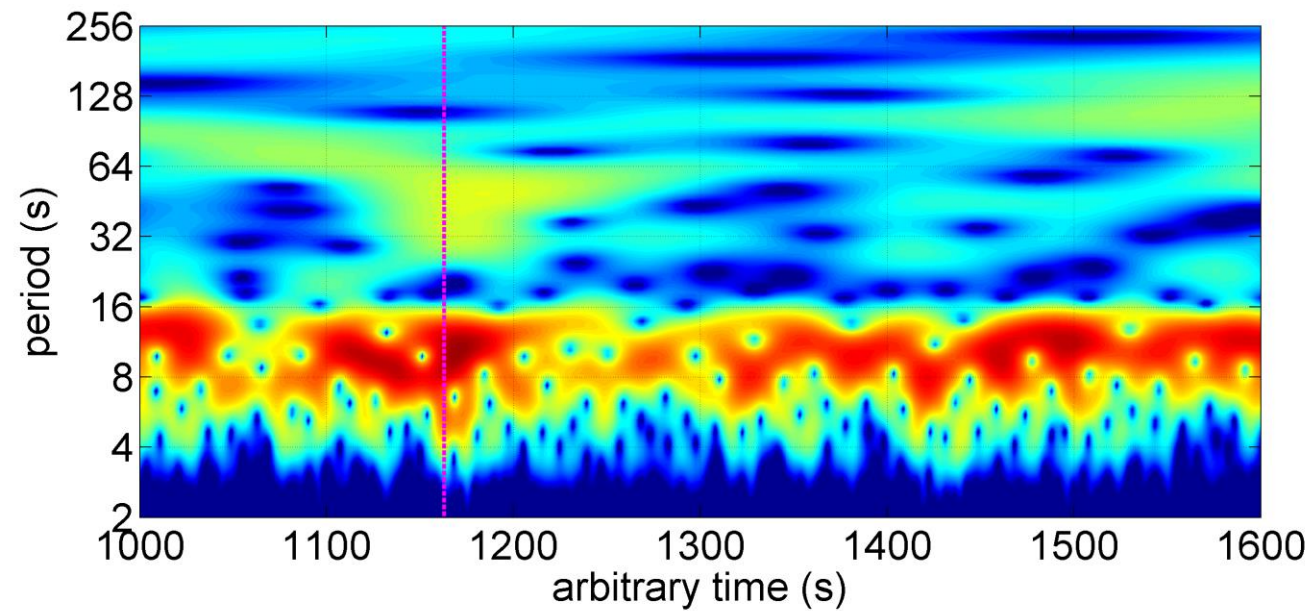
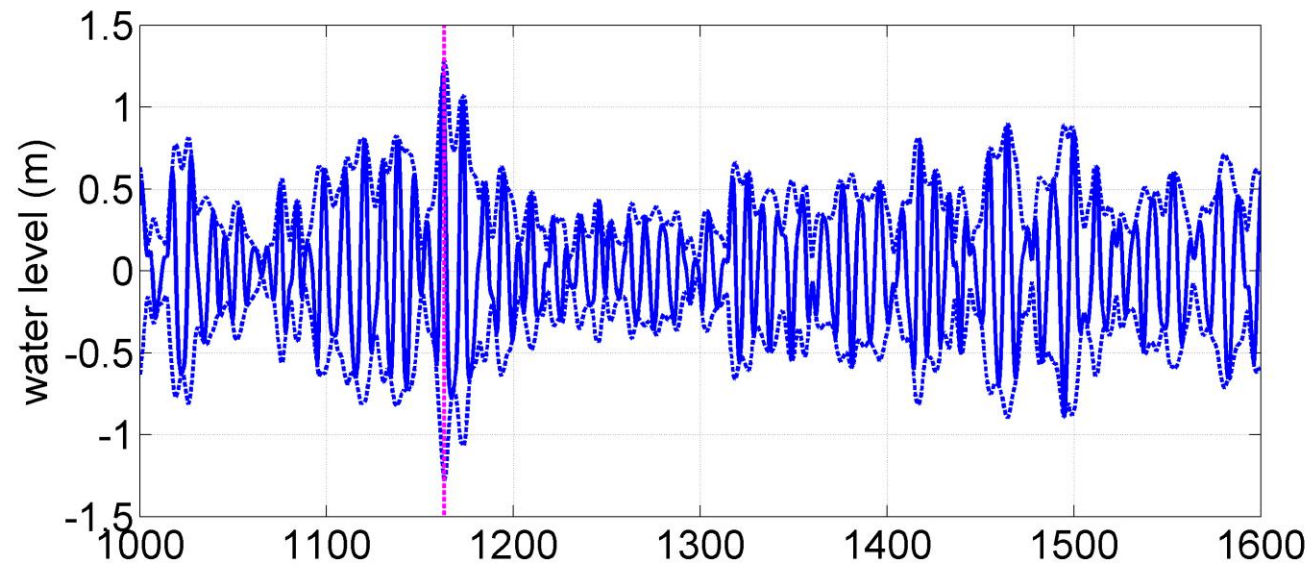


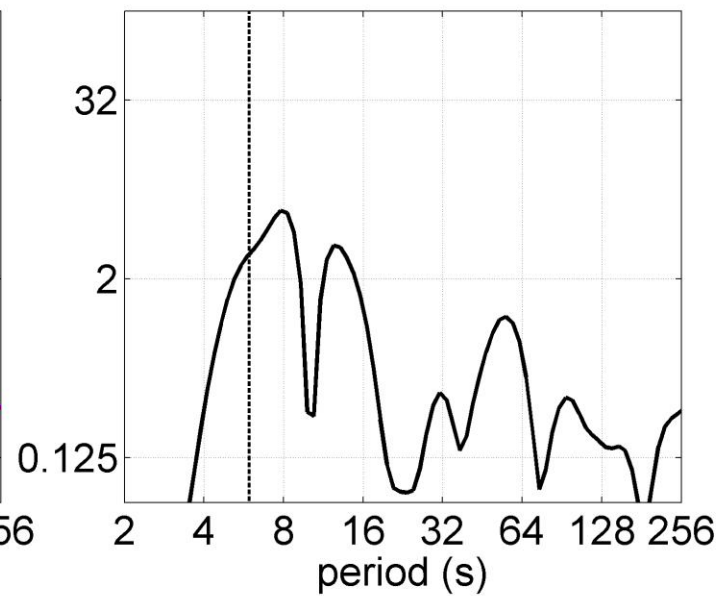
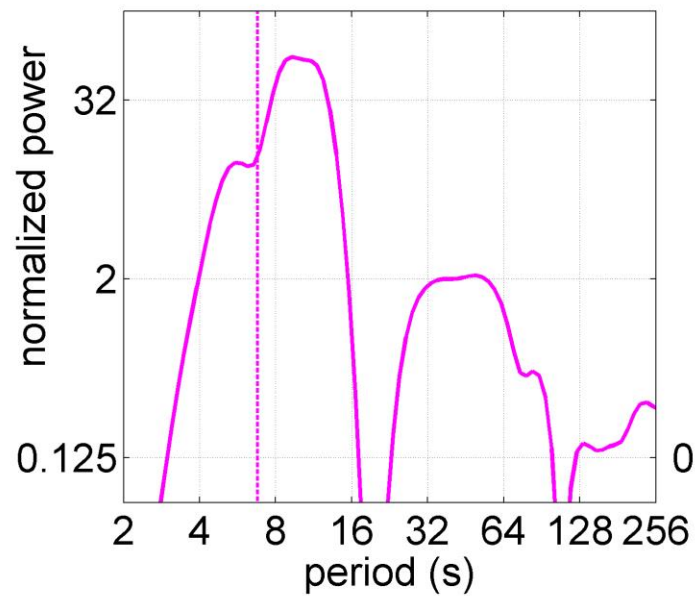
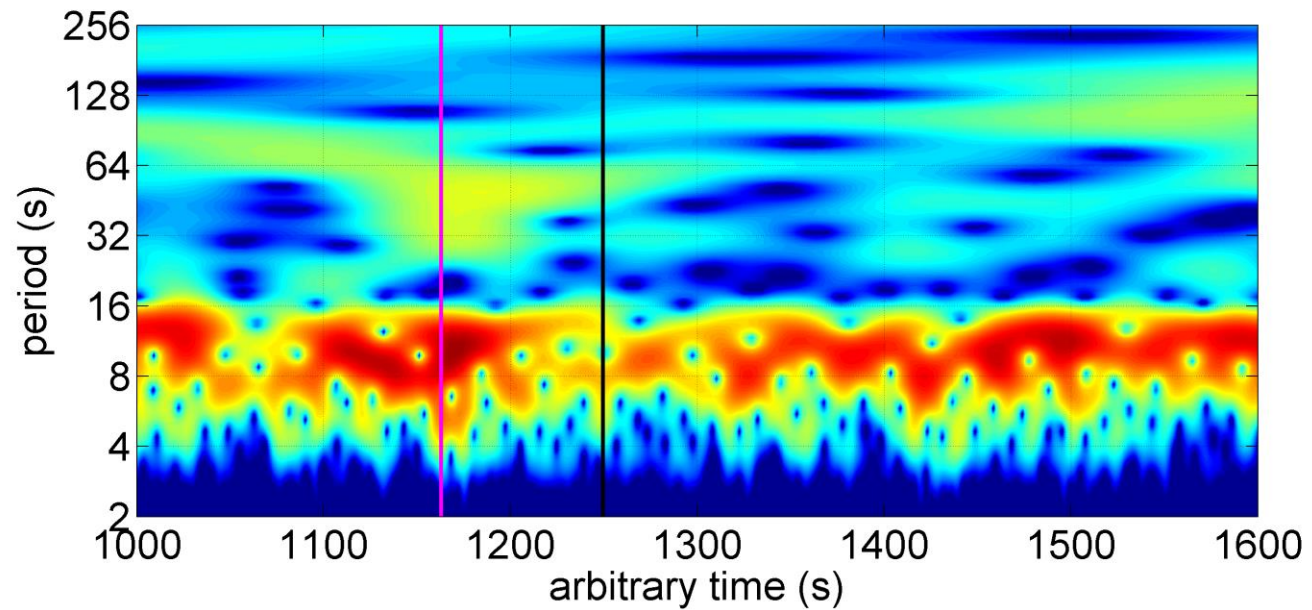
DYNAMIC

$$a\omega^2 > \gamma g$$

$$\gamma = 0.103$$

[7, 8]





# 6

## Appendix: Stokes solution to KdV

$$\eta_t + c_0 \left( 1 + \frac{3}{2} \frac{\eta}{h_0} \right) \eta_x + \gamma \eta_{xxx} = 0$$

$$\frac{\eta}{h_0} = \zeta = \epsilon \zeta_1(\theta) + \epsilon^2 \zeta_2(\theta) + \epsilon^3 \zeta_3(\theta) + O(\epsilon^4)$$

$$\epsilon = a/h_0$$

$$\theta = \kappa x - \omega t$$

[9]

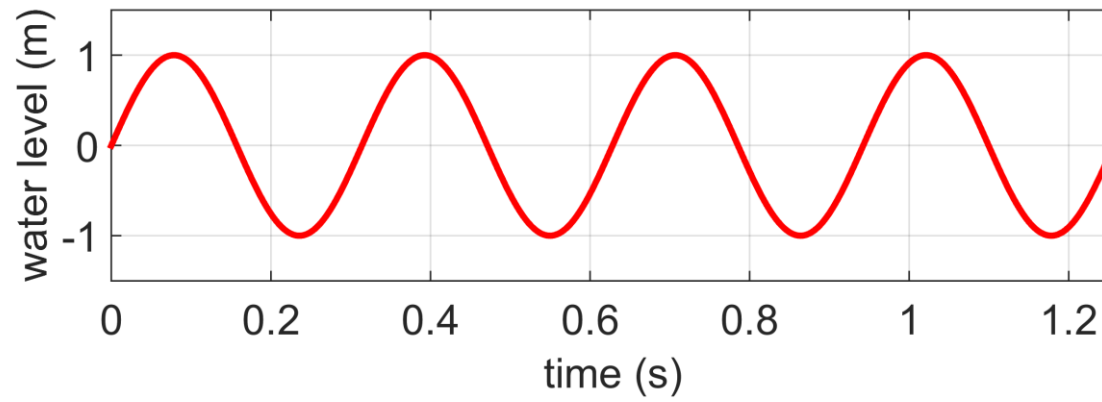
$$\omega = \omega_0 + \epsilon \omega_1 + \epsilon^2 \omega_2 + O(\epsilon^3)$$

$$\zeta_1 = \cos \theta$$

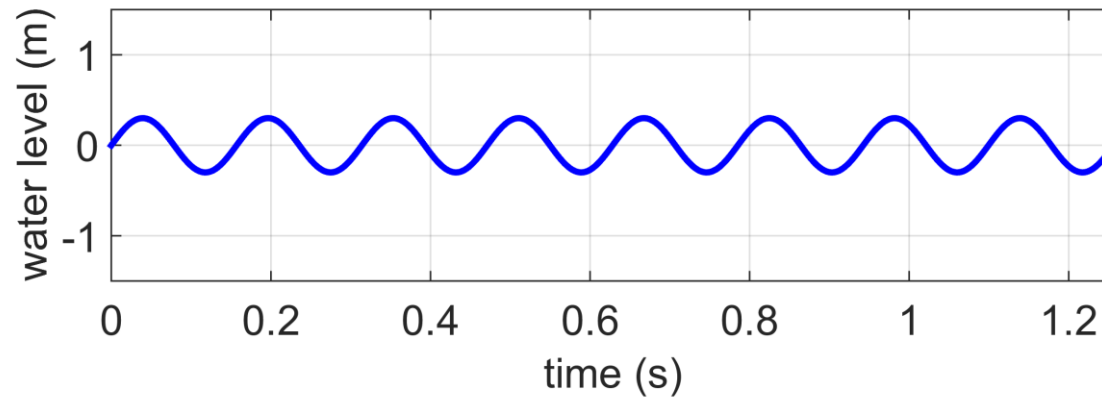
$$\zeta_2 = \frac{c_0}{8\gamma\kappa^2} \cos(2\theta)$$

$$\zeta_3 = \frac{3c_0^2}{256\gamma^2\kappa^4} \cos(3\theta)$$

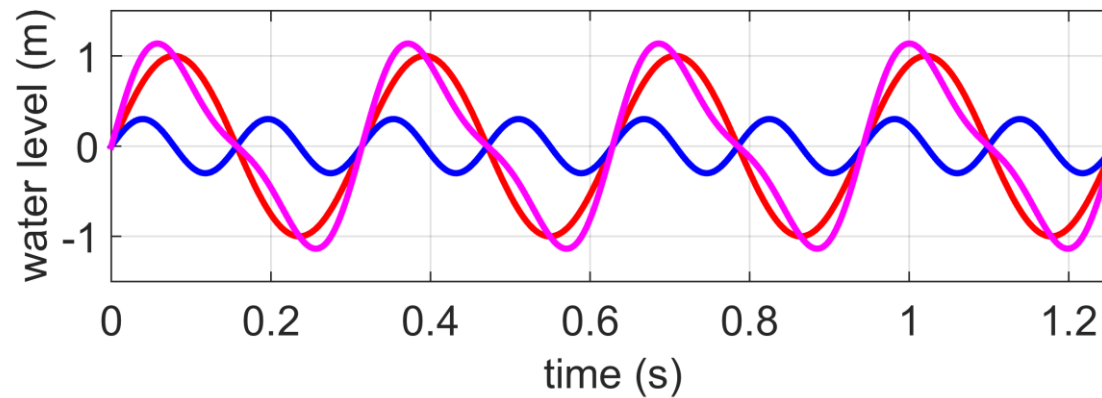




$$\eta_1 = \sin(20t)$$



$$\eta_2 = 0.3 \sin(40t)$$



$$\eta_3 = \eta_1 + \eta_2$$

- [1] Davidson-Arnott, R. (2010), *Introduction to Coastal Processes and Geomorphology*, Cambridge University Press.
- [2] Mei, C. C., M. Stiassnie, and D. K.-P. Yue (2005), Theory and Applications of Ocean Surface Waves. Part 1: Linear Aspects. *Advanced Series in Ocean Engineering*, vol. 23, World Scientific.
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- [6] Torrence, C. and G. P. Compo (1998), A practical guide to wavelet analysis. *Bulletin of the American Meteorological Society*, 79(1), pp. 61-78.
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- [9] Whitham, G. B. (1999), *Linear and Nonlinear Waves*, Pure and Applied Mathematics, John Wiley & Sons, Inc.