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Wavelet as affine group, practical evidence and theoretical hypothesis

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Outline

- Introduction
- Coherent States
- Reconstruction and decomposition
- Utility of the affine group
- Singularity Spectrum
- Practical Evidence
- Conclusions

Introduction

Coherent States

$$[U(a,b)f](x) = |a|^{-1/2}\ f\left(\frac{x-b}{a}\right)$$

$$U(a,b)\ U(a',b') = U(aa',\ b+ab')$$

$$\int_G d\mu(g)\ |\langle f,\ U(g)f\rangle|^2 < \infty$$

$$\int_G d\mu(g)\ \langle h_1, U(g)\tilde{f}\rangle\ \overline{\langle h_2, U(g)\tilde{f}\rangle} = C_{\tilde{f}}\langle h_1, h_2\rangle$$

$$C_{\tilde{f}} = \langle A\tilde{f}, \tilde{f}\rangle$$

$$a^{-2}\ da\,db$$

$$(Af)^\wedge(\xi) = |\xi|^{-1}\ \hat{f}(\xi)$$

[1,2]

But this group structure was not exploited because she went to the discretely labeled wavelet families and these do not correspond to subgroups of the ax+b group.

Meanwhile, the mathematicians…

$$f = C_\psi^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{da\, db}{a^2} \, (T^{\text{wav}} f)(a,b) \; \psi^{a,b}$$

$$C_\psi = 2\pi \int_0^\infty d\xi \; |\xi|^{-1} \; |\hat{\psi}(\xi)|^2 = 2\pi \int_{-\infty}^{0} d\xi \; |\xi|^{-1} \; |\hat{\psi}(\xi)|^2 < \infty$$

[14]

$$f = C_\psi^{-1} \int_0^\infty \frac{da}{a^2} \int_{-\infty}^\infty db\, T^{\text{wav}} f(a,b)\, \psi^{a,b}$$

But using a complex wavelet even for the analysis of real functions may have its advantages…

[14]

Reconstruction and Decomposition

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$\psi_1, \psi_2 \in L^1(\mathbb{R})$ $\psi_2' \in L^2(\mathbb{R})$ $x\psi_2 \in L^1(\mathbb{R})$ $\hat{\psi}_1(0) = 0 = \hat{\psi}_2(0)$

$$f = C_{\psi_1,\psi_2}^{-1} \int \frac{da}{a^2} \int db \; \langle f, \psi_1^{a,b} \rangle \; \psi_2^{a,b}$$

$$f(x) = C_{\psi_1,\psi_2}^{-1} \lim_{\substack{A_1 \to 0 \\ A_2 \to \infty}} \int_{A_1 \le |a| \le A_2} \frac{da}{a^2} \int_{-\infty}^{\infty} db \; \langle f, \psi_1^{a,b} \rangle \; \psi_2^{a,b}(x) .$$

…

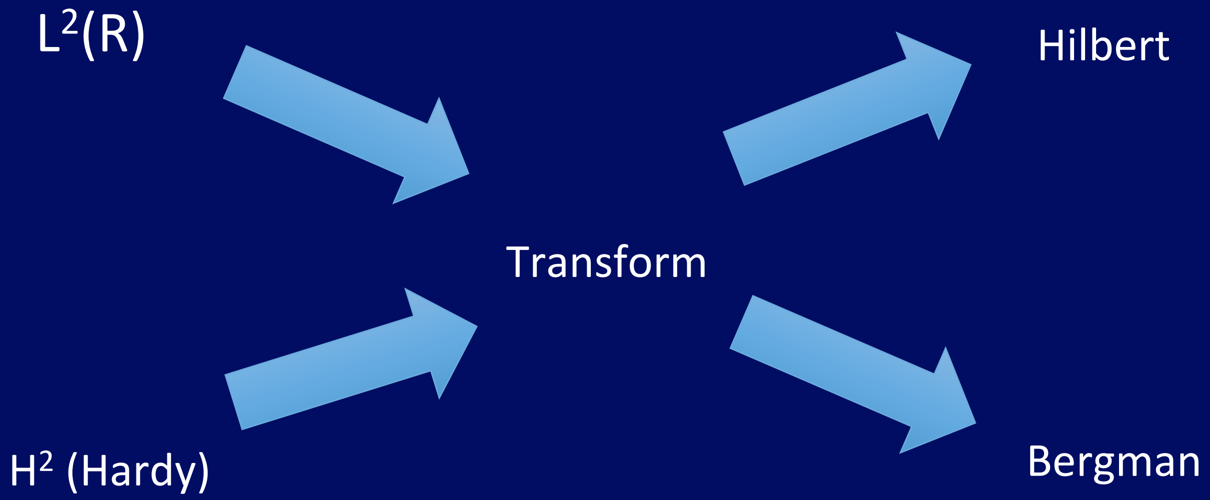
[14]

$$C_{\psi}^{-1} \int \int \frac{da\, db}{a^2} \left|(T^{\mathrm{wav}} f)(a,b)\right|^2 = \int dx \left|f(x)\right|^2$$

The transform maps the space $L^2(R)$ isometrically into a Hilbert space…

In some cases we can choose a subspace of $L^2(R)$, lets say H^2 (Hardy)… The same transform can be interpreted as an isometry from H^2 to the Bergman space

[14]



Then it is obvious that its values at certain discrete families of points completely determines the function… and this is sad

Utility of the Affine Group

"… the utility of the affine group was possible because of the close connection between the affine and the canonical stories while the few distinctions can be used to advantage when appropiate..."

$$[Q, D] = i\hbar Q$$

i) States of a system
ii) Dynamics involves an automorphism among states
iii) The natural inner product interpreted as probably amplitudes

OVERCOMPLETE FAMILY OF STATES

[3-8]

Indeed, modulo linear transformations, the affine variables form the elements of the only two-parameter, non-Abelian Lie algebra.

The affine group gets its name from its realization as the set of affine transformations of the real line given by

$$x \to x' = ax + b$$

Where

$$a \neq 0 \text{ and } b \in \mathbb{R}$$

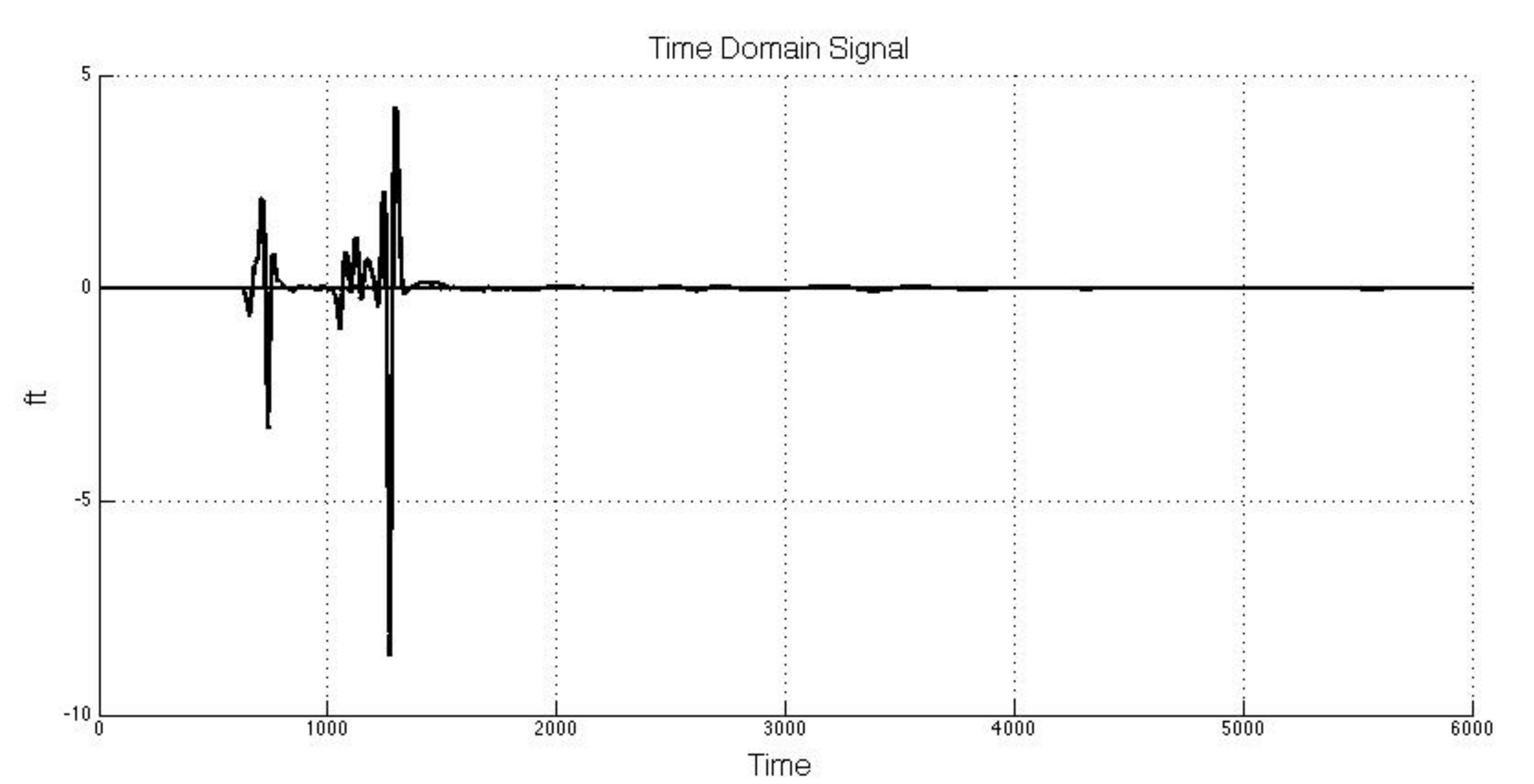
Singularity Spectrum

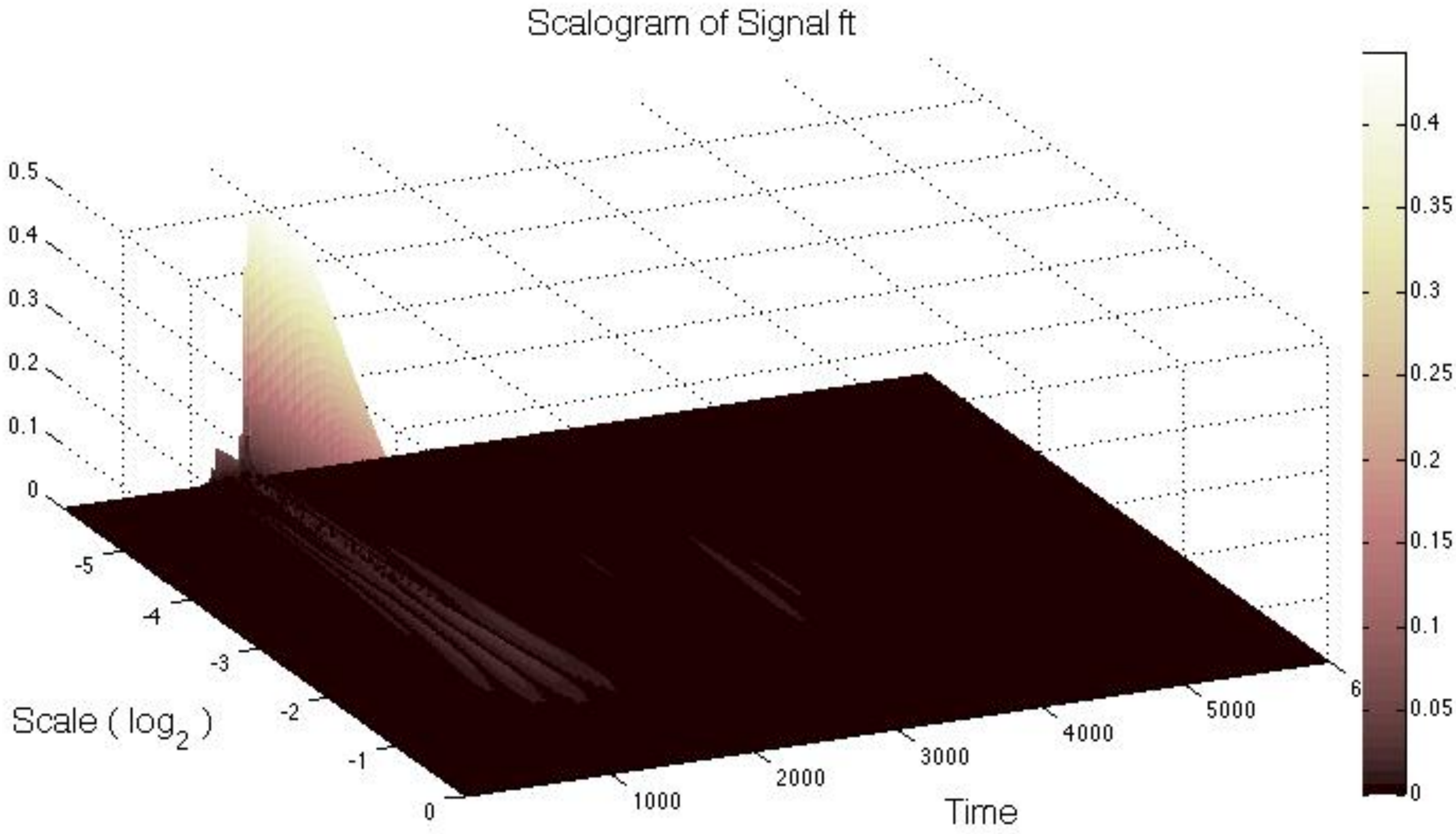
$$W_f(u,s) = < f(t), \psi_{u,s}(t) > = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(t)\, \psi^* \left(\frac{t-u}{s} \right) dt \qquad f(t) = \frac{1}{C_\psi} \int_{0}^{\infty} \int_{-\infty}^{\infty} W_f(u,s) \frac{1}{\sqrt{s}} \psi \left(\frac{t-u}{s} \right) du \, \frac{ds}{s^2}$$

$$\int_{-\infty}^{\infty} t^p \psi(t)\, dt = 0, \qquad p \in \left\{ 0, 1, \ldots, (n-1) \right\}$$

$$W_f(u,s) = f(u) * \bar{\psi}_s(u) \approx W_f[k, s_{i,j}] = \sum_{n=0}^{N-1} f[n] \cdot \bar{\psi}_{s_{i,j}}[n-k] = f[k] \star \bar{\psi}_{s_{i,j}}[k]$$

[12, 13, 14, 20, 21, 22]


Time Domain Signal
5
0
-5
-10
ft
0
1000
2000
3000
4000
5000
6000
Time



Scalogram of Signal ft
0.5
0.4
0.3
0.2
0.1
0
-5
-4
-3
-2
-1
0
Scale (log2)
1000
2000
3000
4000
5000
6
Time
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0

Hölder Distribution

$$|f(t) - P_n(t-u)| \leq K|t-u|^{\alpha}$$

$\alpha \geq 0$ $C > 0$ $\alpha \in (n, n+1)$

$$\int_{-\infty}^{\infty} |\hat{f}(\omega)|(1 + |\omega|^{\alpha})\, d\omega < +\infty$$

Global regularity !

We want local !

$$|\psi(t)| \leq \frac{B_k}{1 + |t|^k}$$

$$\psi_n(t) = (-1)^n \frac{d^n \vartheta(t)}{dt^n}, \quad \vartheta(t) = e^{-t^2/2}$$

$$\bar{\psi}_{n,s}(t) = \frac{(-1)^n}{\sqrt{s}} \frac{d^n \vartheta(-t/s)}{dt^n} = \frac{1}{\sqrt{s}} s^n \left.\frac{d^n \vartheta(\tau)}{d\tau^n}\right|_{\tau = -t/s} = s^n \frac{d^n \bar{\vartheta}_s(t)}{dt^n}$$

[11, 15-17, 22-26]

Hölder Exponent and Continuous Wavelet transform

$$W_{f,n}(u,s) = f(u) * \bar{\psi}_{n,s}(u) = f(u) * s^n \frac{d^n \bar{\vartheta}_s}{dt^n}(u) = s^n \frac{d^n}{dt^n}\left(f * \bar{\vartheta}_s\right)(u)$$

$$|W_f(u,s)| \leq Q s^{\alpha+1/2}\left(1 + \left|\frac{u-\tau}{s}\right|^{\alpha}\right), \quad \forall (u,s) \in \mathbb{R} \times \mathbb{R}^+$$

$$|W_f(u,s)| \leq Q s^{\alpha+1/2}\left(1 + \left|\frac{u-\tau}{s}\right|^{\beta}\right), \quad \forall (u,s) \in \mathbb{R} \times \mathbb{R}^+$$

[17, 18, 19]

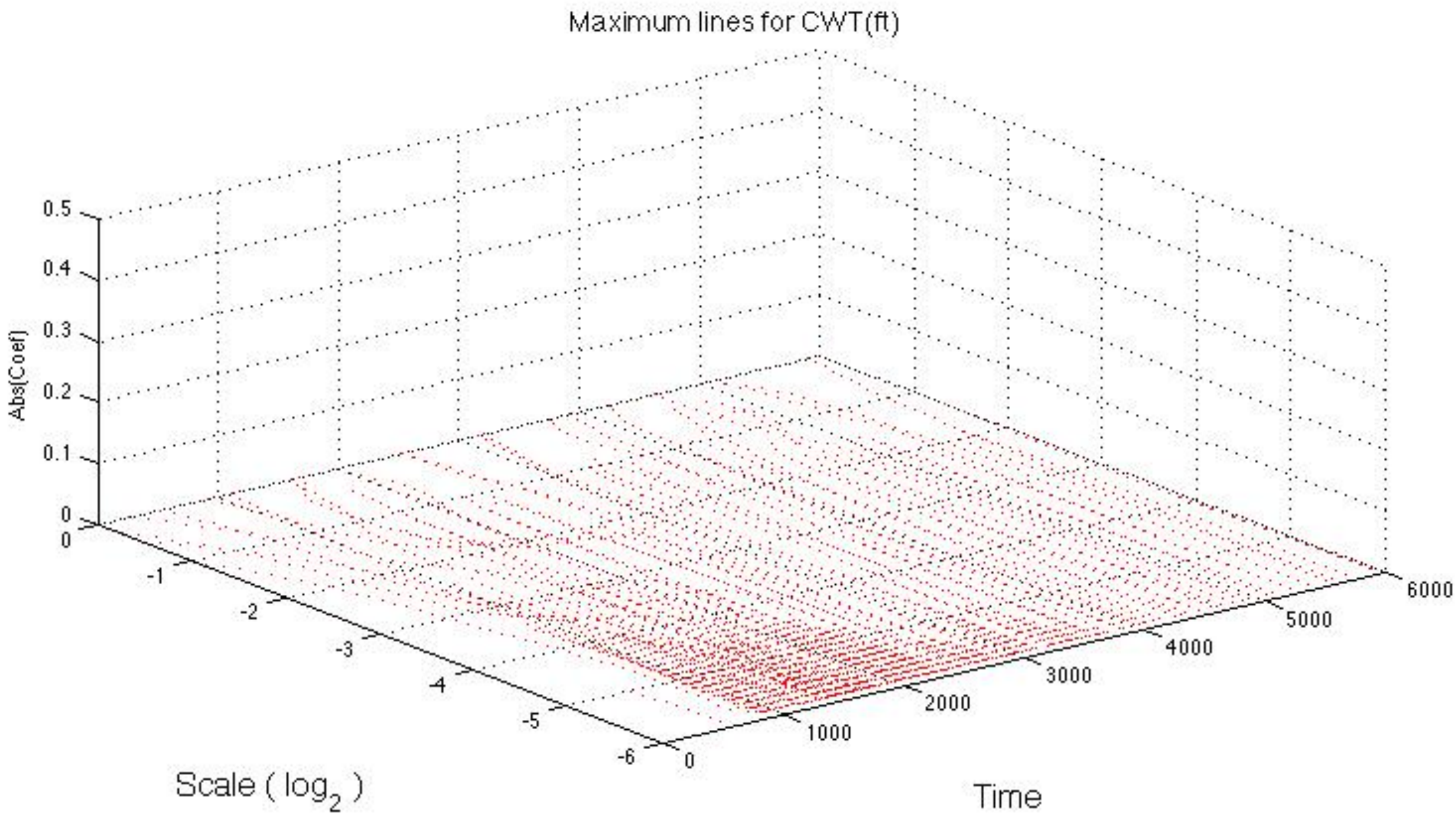
Wavelet transform Maximus Modulus

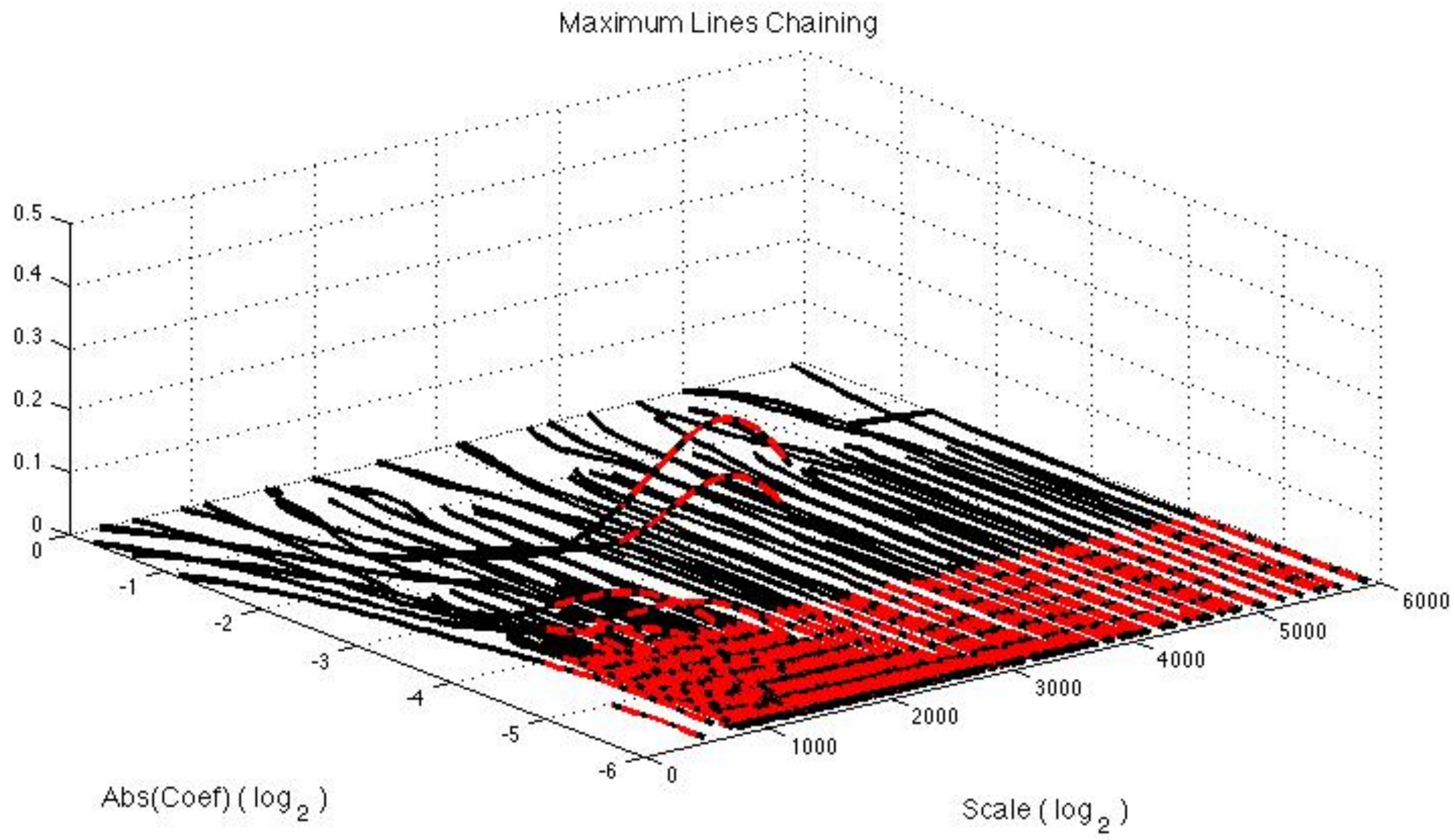
$$|W_f(u,s)| \leq M s^{\alpha+1/2}$$

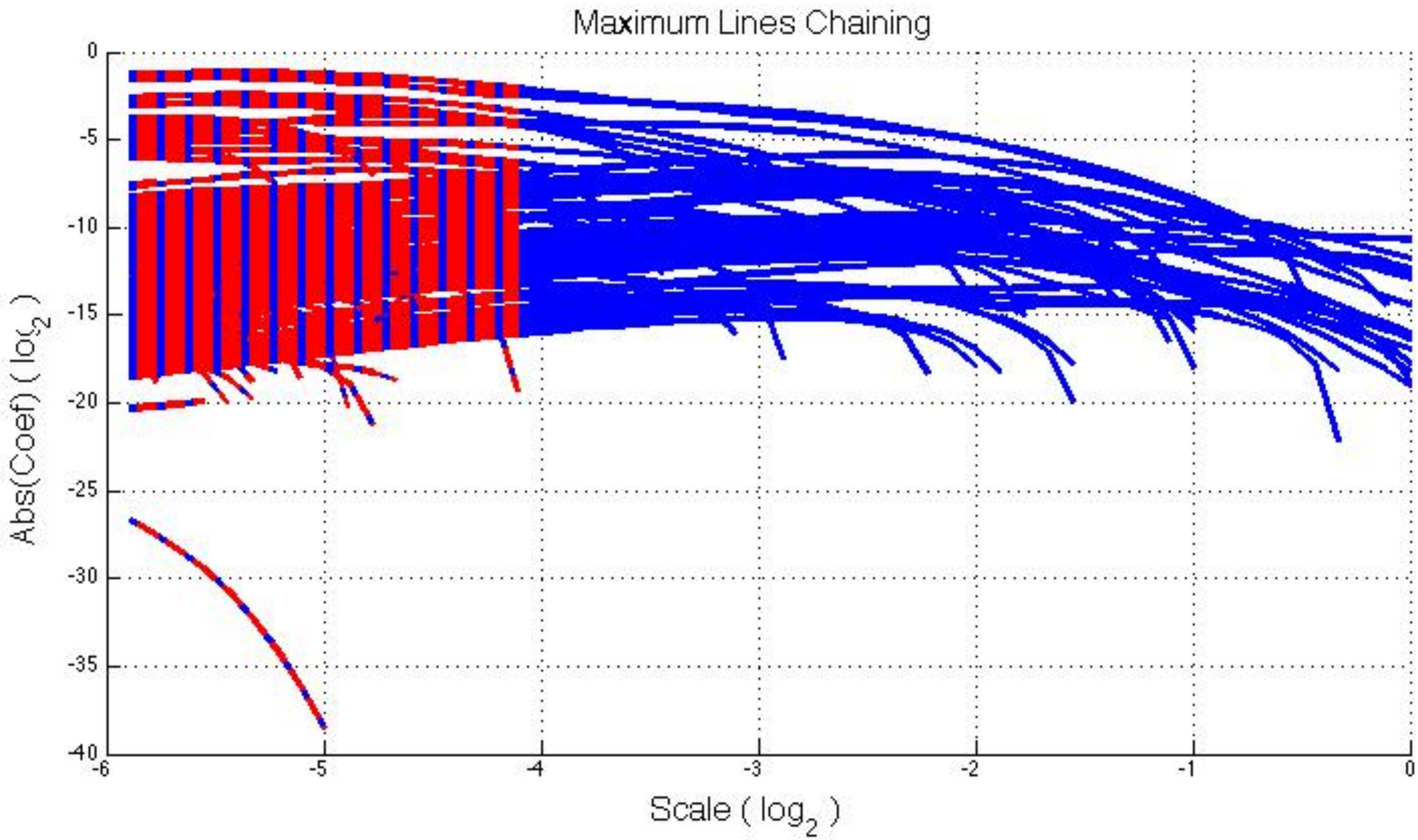
$$\log_2(|w_f(u,s)|) \approx \log_2(M) + \log_2(\alpha + 1/2)\log_2(s)$$

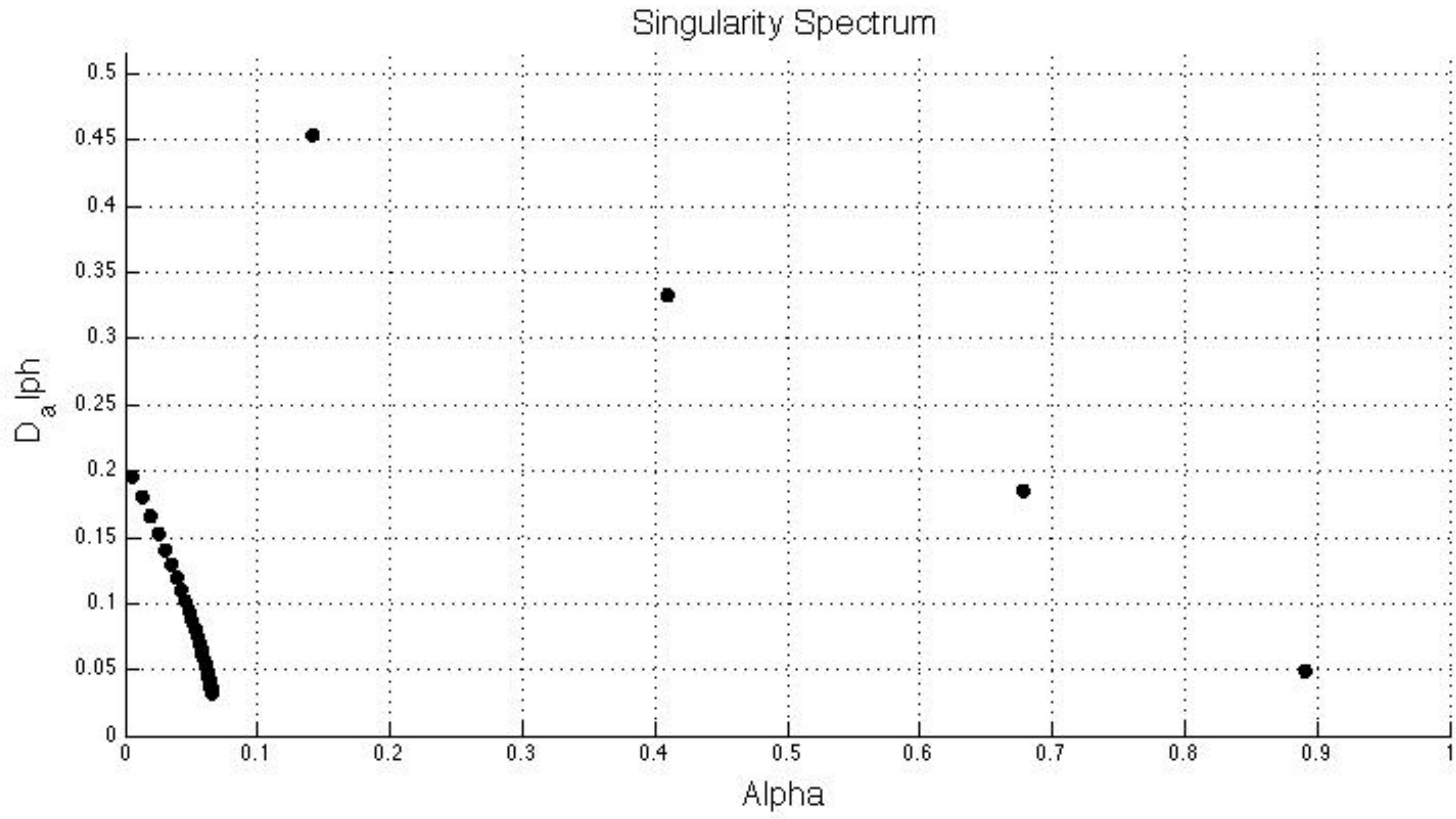
$$WTMM = \left\{ (u_0, s_0) \in (\mathbb{R}, \mathbb{R}^+), \ \left. \frac{\partial |W_f(u,s)|}{\partial u} \right|_{u=u_0, s=s_0} = 0 \right\}$$

[20]

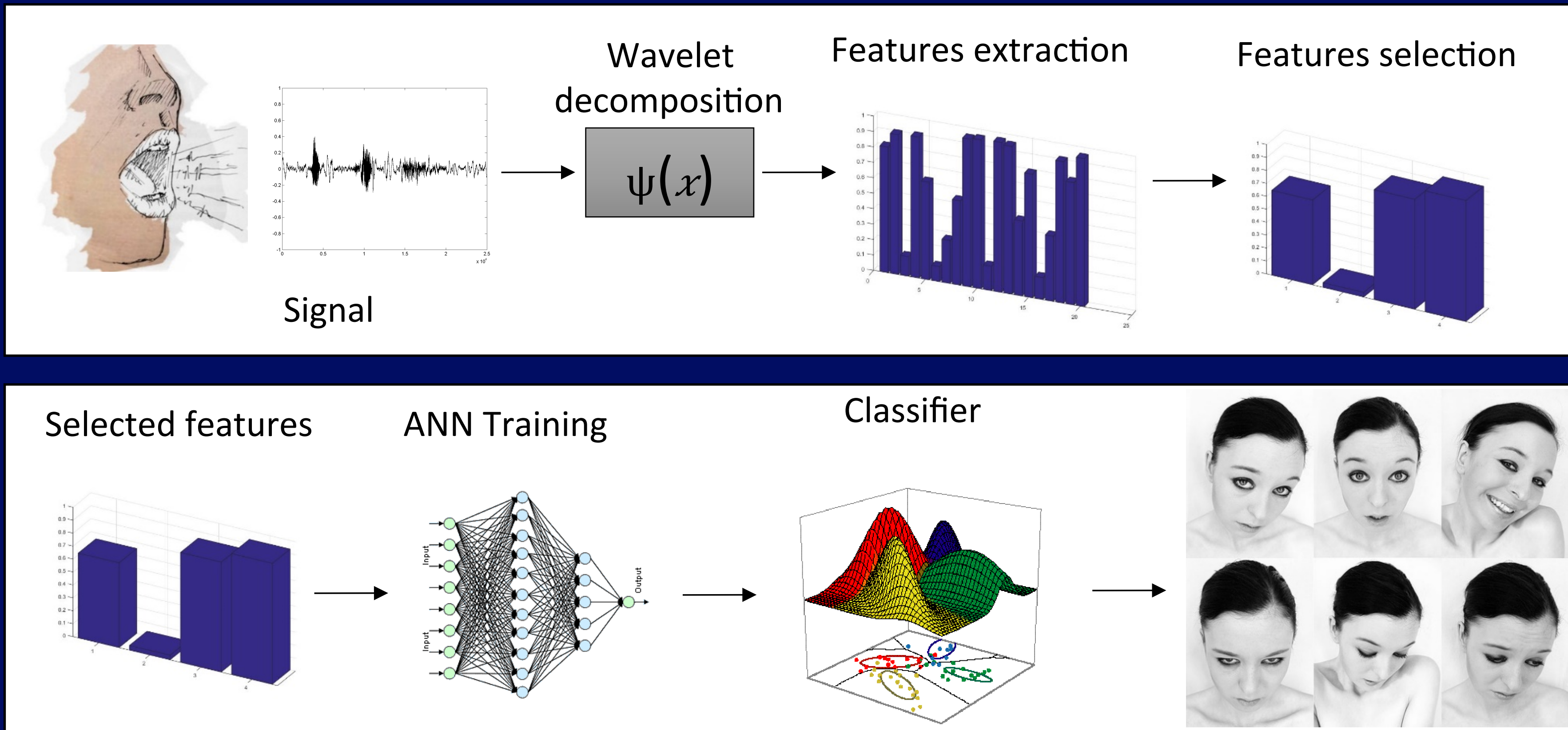

Maximum lines for CWT(ft)
Abs(Coef)
0.5
0.4
0.3
0.2
0.1
0
0
-1
-2
-3
-4
-5
-6
Scale (log_2)
0
1000
2000
3000
4000
5000
6000
Time


Maximum Lines Chaining
0.5
0.4
0.3
0.2
0.1
0
0
-1
-2
-3
-4
-5
-6
0
1000
2000
3000
4000
5000
6000
Abs(Coef) (log 2)
Scale (log 2)


Maximum Lines Chaining
Abs(Coef) (log2)
Scale (log2)
0
-5
-10
-15
-20
-25
-30
-35
-40
-6
-5
-4
-3
-2
-1
0


Singularity Spectrum
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Alpha
D_a lph

Practical Evidence


Wavelet
decomposition
Features extraction
Features selection
$\psi(x)$
Signal
Selected features
ANN Training
Classifier
Input
Output

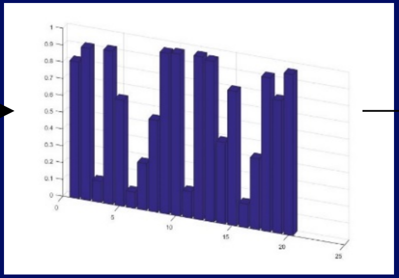
[27-30]

Wavelet Decomposition | 10 levels aprox -detail | Features Extraction | Técnica de selección de características

db1
db6
db8
db10

$\psi(x)$

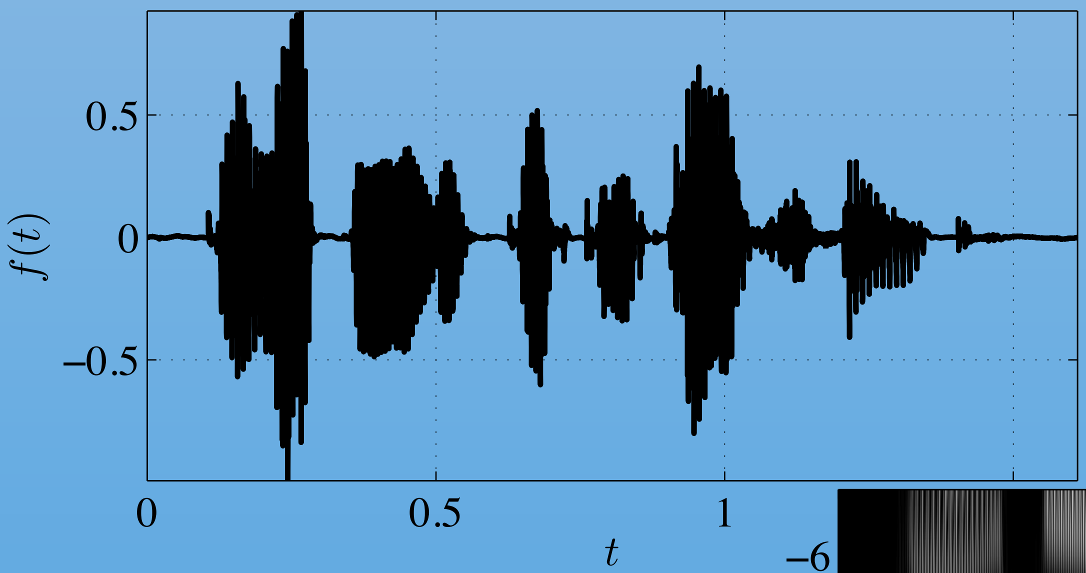
$$\begin{pmatrix} V_1 & W_1 \\ \vdots & \vdots \\ \vdots & \vdots \\ V_{10} & W_{10} \end{pmatrix}$$

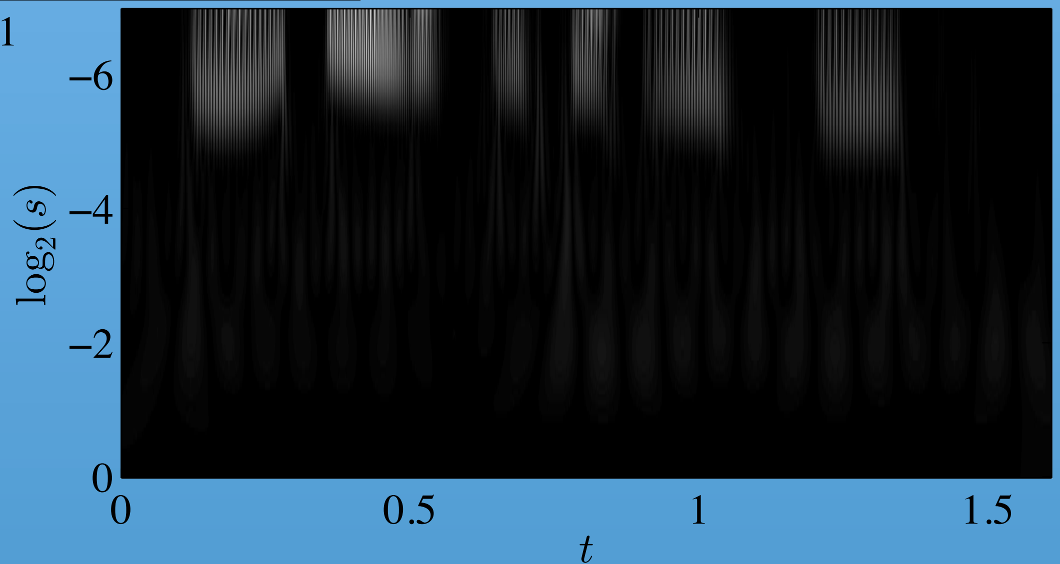


- Max and mins
- Mean and mediana
- Interq mean
- Standard deviation
- Statistic assymetry
- Kurtosis

Mean difference test between pairs

EMOTIONS	*BOREDOM*	*DISGUST*	*HAPPINESS*	*FEAR*	*NEUTRAL*	*ANGER*	*SADNESS*
BOREDOM	**89.74%**	0%	0%	1.28%	6.41%	1.28%	1.28%
DISGUST	2.22%	**91.11%**	4.44%	0%	2.22%	0%	0%
HAPPINESS	0%	2.86%	**85.71%**	2.86%	1.43%	7.14%	0%
FEAR	1.45%	1.45%	1.45%	**92.75%**	2.9%	0%	0%
NEUTRAL	6.25%	0%	0%	2.5%	**91.25%**	0%	0%
ANGER	0%	0.79%	4.76%	1.59%	0%	**92.86%**	0%
SADNESS	1.67%	0%	0%	1.67%	3.33%	0%	**93.33%**





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Conclusions

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